

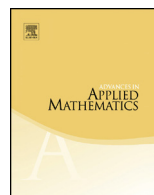


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# Variations of the Poincaré series for affine Weyl groups and $q$ -analogues of Chebyshev polynomials

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## ABSTRACT

Let  $(W, S)$  be a Coxeter system and write  $P_W(q)$  for its Poincaré series. Lusztig has shown that the quotient  $P_W(q^2)/P_W(q)$  is equal to a certain power series  $L_W(q)$ , defined by specializing one variable in the generating function recording the lengths and absolute lengths of the involutions in  $W$ . The simplest inductive method of proving this result for finite Coxeter groups suggests a natural bivariate generalization  $L_W^J(s, q) \in \mathbb{Z}[[s, q]]$  depending on a subset  $J \subset S$ . This new power series specializes to  $L_W(q)$  when  $s = -1$  and is given explicitly by a sum of rational functions over the involutions which are minimal length representatives of the double cosets of the parabolic subgroup  $W_J$  in  $W$ . When  $W$  is an affine Weyl group, we consider the renormalized power series  $T_W(s, q) = L_W^J(s, q)/L_W(q)$  with  $J$  given by the generating set of the corresponding finite Weyl group. We show that when  $W$  is an affine Weyl group of type  $A$ , the power series  $T_W(s, q)$  is actually a polynomial in  $s$  and  $q$  with nonnegative coefficients, which turns out to be a  $q$ -analogue recently studied by Cigler of the Chebyshev polynomials of the first kind, arising in a completely different context.

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### 1. Introduction

#### 1.1. Background and motivation

Let  $(W, S)$  be a Coxeter system with length function  $\ell : W \rightarrow \mathbb{N}$ . The *Poincaré series* of  $(W, S)$  is the formal power series (in an indeterminate  $q$ ) given by

$$P_W(q) = \sum_{w \in W} q^{\ell(w)} \in \mathbb{Z}[[q]].$$

This power series is well-defined if and only if the rank of  $(W, S)$  is finite, and in this work, therefore, we require all Coxeter systems  $(W, S)$  to have  $|S| < \infty$ . If  $W$  is finite then  $P_W(q)$  is obviously a polynomial, and in general  $P_W(q)$  is always a rational power series; see [15,28].

Lusztig [21,22] has introduced an interesting analogue of the Poincaré series defined in terms of the twisted involutions in a Coxeter group, and our main object of study here is a natural bivariate generalization of this power series. To motivate its definition, we review some relevant information from [21,22].

To begin, let  $\text{Aut}(W, S)$  denote the group of automorphisms of  $W$  preserving  $S$ , and fix an involution (that is, a self-inverse automorphism)  $*$   $\in$   $\text{Aut}(W, S)$ . We denote the action of  $*$  on elements  $w \in W$  by  $w^*$ , and write

$$\mathbf{I}_* = \mathbf{I}_*(W) \stackrel{\text{def}}{=} \{w \in W : w^{-1} = w^*\}$$

for the corresponding set of *twisted involutions* in  $W$ . The “twisted” analogue of  $P_W(q)$  is the formal power series

$$L_{W,*}(q) = \sum_{w \in \mathbf{I}_*} q^{\ell(w)} \left(\frac{q-1}{q+1}\right)^{\ell^*(w)} \in \mathbb{Z}[[q]] \tag{1.1}$$

where on the right side  $\ell^*$  denotes the *twisted absolute length function* defined by Hultman in [14], which is characterized explicitly as the unique map  $\mathbf{I}_* \rightarrow \mathbb{N}$  such that

- (a)  $\ell^*(1) = 0$ ;
- (b)  $\ell^*$  is constant on  $*$ -twisted conjugacy classes, i.e.,  $\ell^*(s w s^*) = \ell^*(w)$  for all  $s \in S$ ;
- (c)  $\ell^*(ws) - \ell^*(w) = \ell(ws) - \ell(w)$  whenever  $s \in S$  and  $w \in \mathbf{I}_*$  are such that  $ws \in \mathbf{I}_*$ .

Note in (c) that  $ws \in \mathbf{I}_*$  if and only if  $ws = s^*w$ . The function  $\ell^*$  is the same as the map denoted  $\phi$  in [21,22]. Lusztig’s paper [21, §5.8] appears to be the first place in the literature where the power series (1.1) is considered, and for this reason we denote it by the letter  $L$ .

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