

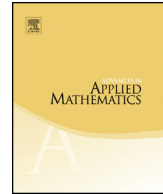


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Generalized Pascal triangle for binomial coefficients of words



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ABSTRACT

We introduce a generalization of Pascal triangle based on binomial coefficients of finite words. These coefficients count the number of times a word appears as a subsequence of another finite word. Similarly to the Sierpiński gasket that can be built as the limit set, for the Hausdorff distance, of a convergent sequence of normalized compact blocks extracted from Pascal triangle modulo 2, we describe and study the first properties of the subset of $[0, 1] \times [0, 1]$ associated with this extended Pascal triangle modulo a prime p .

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1. Introduction

Pascal triangle and the corresponding Sierpiński gasket are well studied objects (see, for instance, [15] for a survey). They exhibit self-similarity features and have connections with dynamical systems, cellular automata, number theoretic questions and automatic

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sequences [2,1,5,10,13]. In this paper, we will consider a variation of these two objects by extending binomial coefficients to the free monoid A^* where A is a finite alphabet.

Let us start with basic combinatorial definitions. A *finite word* is simply a finite sequence of letters belonging to a finite set called *alphabet*. In combinatorics on words, one can introduce the binomial coefficient $\binom{u}{v}$ of two finite words u and v which is the number of times v occurs as a subsequence of u (meaning as a “scattered” subword). As an example, we consider two particular words over $\{0, 1\}$ and

$$\binom{101001}{101} = 6.$$

Indeed, if we index the letters of the first word $u_1u_2 \cdots u_6 = 101001$, we have

$$u_1u_2u_3 = u_1u_2u_6 = u_1u_4u_6 = u_1u_5u_6 = u_3u_4u_6 = u_3u_5u_6 = 101.$$

Observe that this concept is a natural generalization of the binomial coefficients of integers. For a single letter alphabet $\{a\}$, we have

$$\binom{a^m}{a^n} = \binom{m}{n}, \quad \forall m, n \in \mathbb{N} \tag{1}$$

where a^m denotes the concatenation of m letters a . For more on these binomial coefficients, see for instance [11, Chap. 6].

In this paper, we are interested in Pascal triangle obtained when considering binomial coefficients of words. To define such a triangular array, we will consider all the words over a finite alphabet and we order them by genealogical ordering (i.e., first by length, then by the classical lexicographic ordering for words of the same length assuming $0 < 1$). For the sake of simplicity, we will mostly discuss the case of a 2-letter alphabet $\{0, 1\}$ and to relate these words to base-2 expansions of integers, we will assume without loss of generality that the non-empty words start with 1. (If leading zeroes were allowed, then different words could represent the same integer.)

Definition 1. We let $\text{rep}_2(n)$ denote the greedy base-2 expansion of $n \in \mathbb{N}_{>0}$ starting with 1 where the notation $\mathbb{N}_{>0}$ stands for the set of all positive integers. We set $\text{rep}_2(0)$ to be the empty word denoted by ε . Let $L = \{\varepsilon\} \cup 1\{0, 1\}^*$ be the set of base-2 expansions of the integers. We let L_n denote the set of words of length at most n belonging to L , i.e.,

$$L_n = (\{\varepsilon\} \cup 1\{0, 1\}^*) \cap \{0, 1\}^{\leq n}.$$

Note that $\#L_n = 2^n$ for all $n \geq 0$.

The first few values in the generalized Pascal triangle T that we will deal with are given in Table 1. These values correspond to the words $\varepsilon, 1, 10, 11, 100, 101, 110, 111$.

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