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The excluded minors for the class of matroids that are graphic or bicircular lift ☆

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ABSTRACT

Bicircular lift matroids are a class of matroids defined on the edge set of a graph. For a given graph G, the circuits of its bicircular lift matroid $L(G)$ are the edge sets of those subgraphs of G that contain at least two cycles, and are minimal with respect to this property. For each cycle C of G, since $L(G)/C$ is graphic and most graphic matroids are not bicircular lift, the class of bicircular lift matroids is not minor-closed. In this paper, we prove that the class of matroids that are graphic or bicircular lift has a finite list of excluded minors.

1. Introduction

We assume that the reader is familiar with fundamental definitions in matroid and graph theory. All definitions in matroid theory that are used but not defined in the paper follow from Oxley's book [3]. For a graph G, a set $X \subseteq E(G)$ is a *cycle* if $G|X$ is a connected 2-regular graph, bicircular lift matroids are a class of matroids defined on the edge set of a graph. For a given graph G, the circuits of its bicircular lift matroid $L(G)$ are the edge sets of those subgraphs of G that contain at least two cycles, and

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are minimal with respect to this property. That is, the circuits of $L(G)$ consist of the edge sets of two edge-disjoint cycles with at most one common vertex, or three internally disjoint paths between a pair of distinct vertices.

Bicircular lift matroids are a special class of lift matroids that arises from biased graphs, where biased graphs and its lift matroids were introduced by Zaslavsky in [8,9]. Let BL denote the class of bicircular lift matroids. For each cycle C of G, since $L(G)/C$ is graphic and most graphic matroids are not in BL by the following Lemma 4.4, this class BL is not minor-closed. But the union of BL and the class of graphic matroids is a minor-closed class. Let $\overline{BL}$ denote this class. Irene Pivotto [4] conjectured

Conjecture 1.1. *The class $\overline{BL}$ has a finite list of excluded minors.*

In this paper, we prove that the conjecture is true. In fact, we prove a stronger result.

Theorem 1.2. *Let M be an excluded minor of $\overline{BL}$. Then*

- *either M is a direct sum of the uniform matroid $U_{2,4}$ and a loop, or*
- *M is 3-connected with $r(M) \leq 11$ and with $|E(M)| \leq 224$.*

In the rest of paper, we always let M be an excluded minor of $\overline{BL}$. The paper is organized as follows. Some related definitions and basic results are given in Section 2. In Section 3, we prove that when M is not connected, M is a direct sum of the uniform matroid $U_{2,4}$ and a loop. In Section 4, we prove that if M is connected then M is 3-connected. In Section 5, we prove that if M is 3-connected then $r(M) \leq 11$ and $|E(M)| \leq 224$.

Unfortunately, the number of matroids with rank at most 11 and size at most 224 is massive. There are too many matroids! The bound is outside what we are able to check with a computer. The search space is just too large.

2. Preliminaries

Let G be a graph. Set $|G| := |V(G)|$. For a vertex v of G, let $st_G(v)$ denote the set of all edges adjacent with v. An edge of G is a *link* if its end-vertices are distinct; otherwise it is a *loop*. Let $\mathrm{loop}(G)$ be the set consisting of loops of G. We say that G is *2-edge-connected* if each edge of G is contained in some cycle. A graph obtained from graph G with some edges of G replaced by internally disjoint paths is a *subdivision* of G.

Let $e, f \in E(G)$. If $\{e, f\}$ is a cycle, then e and f are *a parallel pair*. A *parallel class* of G is a maximal subset P of $E(G)$ such that any two members of P are a parallel pair and no member is a loop. Moreover, if $|P| \geq 2$ then P is *non-trivial*; otherwise P is *trivial*. Let $\mathrm{si}(G)$ denote the graph obtained from G by deleting all loops and all but one distinguished element of each non-trivial parallel class. Obviously, the graph we obtain