

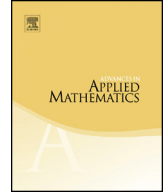


ELSEVIER

Contents lists available at ScienceDirect

Advances in Applied Mathematics

www.elsevier.com/locate/yaama



Parabolic Kazhdan–Lusztig polynomials for quasi-minuscule quotients



Francesco Brenti^{a,*}, Pietro Mongelli^b, Paolo Sentinelli^{a,1}

^a *Dipartimento di Matematica, Università di Roma “Tor Vergata”, Via della Ricerca Scientifica 1, 00173 Roma, Italy*

^b *Dipartimento di Matematica, Università di Roma “La Sapienza”, Piazzale Aldo Moro, Roma, Italy*

ARTICLE INFO

Article history:

Received 7 August 2015

Accepted 19 January 2016

Available online 8 March 2016

MSC:

primary 05E10

secondary 20F55

Keywords:

Kazhdan–Lusztig polynomial

Quasi-minuscule quotient

Weyl group

Combinatorics

ABSTRACT

We study the parabolic Kazhdan–Lusztig polynomials for the quasi-minuscule quotients of Weyl groups. We give explicit closed combinatorial formulas for the parabolic Kazhdan–Lusztig polynomials of type q . Our study implies that these are always either zero or a monic power of q , and that they are not combinatorial invariants. We conjecture a combinatorial interpretation for the parabolic Kazhdan–Lusztig polynomials of type -1 .

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

In 1979, Kazhdan and Lusztig [16] introduced a family of polynomials, indexed by pairs of elements in a Coxeter group W , which plays an important role in various areas

* Corresponding author.

E-mail addresses: brenti@mat.uniroma2.it (F. Brenti), mongelli@mat.uniroma1.it (P. Mongelli), paolosentinelli@gmail.com (P. Sentinelli).

¹ Current address: Departamento de Matemáticas, Universidad de Chile, Casilla 653, Las Palmeras 3425, Santiago, Chile.

of mathematics, including the algebraic geometry and topology of Schubert varieties and representation theory (see, e.g., [1] p. 171 and the references cited there). These celebrated polynomials are now known as the Kazhdan–Lusztig polynomials of W (see, e.g., [1] or [14]). In 1987, Deodhar [7] developed an analogous theory for the parabolic setup. Given any parabolic subgroup W_J in a Coxeter system (W, S) , Deodhar introduced two Hecke algebra modules (one for each of the two roots q and -1 of the polynomial $x^2 - (q-1)x - q$) and two families of polynomials $\{P_{u,v}^{J,q}(q)\}_{u,v \in W^J}$ and $\{P_{u,v}^{J,-1}(q)\}_{u,v \in W^J}$ indexed by pairs of elements of the set of minimal coset representatives W^J . These polynomials are the parabolic analogues of the Kazhdan–Lusztig polynomials: while they are related to their ordinary counterparts in several ways (see, e.g., §2 and [7], Proposition 3.5), they also play a direct role in several areas such as the geometry of partial flag manifolds [15], the theory of Macdonald polynomials [12,13], tilting modules [24,25], generalized Verma modules [5], canonical bases [10,28], the representation theory of the Lie algebra $\mathfrak{gl}_n$ [20], quantized Schur algebras [29], quantum groups [8], and physics (see, e.g., [11], and the references cited there). The computation of these polynomials is a very difficult task. Although a geometric interpretation for the (ordinary and parabolic) polynomials exists (see [17] and [15]) in the case of Weyl groups and an algebraic interpretation exists for the ordinary ones [9] for all Coxeter systems, there are very few explicit formulas for them (see, e.g., [1], p. 172, and the references cited there).

The purpose of this work is to study the parabolic Kazhdan–Lusztig polynomials for the quasi-minuscule quotients of Weyl groups. These quotients possess noteworthy combinatorial and geometric properties (see, e.g., [18] and [27]). The parabolic Kazhdan–Lusztig polynomials for the minuscule quotients have been computed in [19,2–4]. In this work we turn our attention to the quasi-minuscule quotients that are not minuscule (also known as (co-)adjoint quotients). More precisely, we obtain closed combinatorial formulas for the parabolic Kazhdan–Lusztig polynomials of type q of these quotients for the classical Weyl groups. Our results imply that these are always either zero or a monic power of q for all quasi-minuscule quotients, and that they are not combinatorial invariants. For the parabolic Kazhdan–Lusztig polynomials of type -1 we conjecture explicit combinatorial interpretations.

The organization of the paper is as follows. In Section 2 we recall some definitions, notation and results that are used in the sequel. In Section 3 we give combinatorial descriptions of the quasi-minuscule quotients of classical Weyl groups. In Section 4 we give combinatorial formulas for the parabolic Kazhdan–Lusztig polynomials of type q of (co-)adjoint quotients of classical Weyl groups. Our results imply that these polynomials are always either zero or a monic power of q for all quasi-minuscule quotients, and that they are not combinatorial invariants. In Section 5 we derive some consequences of our results for the classical Kazhdan–Lusztig polynomials. Finally, in Section 6 we present our conjectured combinatorial interpretations for the parabolic Kazhdan–Lusztig polynomials of type -1 of the (co-)adjoint quotients of classical Weyl groups, and the evidence that we have in their favor.

Download English Version:

<https://daneshyari.com/en/article/4624515>

Download Persian Version:

<https://daneshyari.com/article/4624515>

[Daneshyari.com](https://daneshyari.com)