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The binary matroids whose only odd circuits are triangles

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A R T I C L E I N F O

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This paper generalizes a graph-theoretical result of Maffray to binary matroids. In particular, we prove that a connected simple binary matroid M has no odd circuits other than triangles if and only if M is affine, M is $M(K_4)$ or F_7, or M is the cycle matroid of a graph consisting of a collection of triangles all of which share a common edge. This result implies that a 2-connected loopless graph G has no odd bonds of size at least five if and only if G is Eulerian or G is a subdivision of either K_4 or the graph that is obtained from a cycle of parallel pairs by deleting a single edge.

1. Introduction

For each $n \geq 1$, let $K'_{2,n}$ be the graph that is obtained from $K_{2,n}$ by adding an edge joining the vertices in the two-vertex class (see Fig. 1). Maffray [5, Theorem 2] proved the following result.

Theorem 1.1. *A 2-connected simple graph G has no odd cycles of length exceeding three if and only if*

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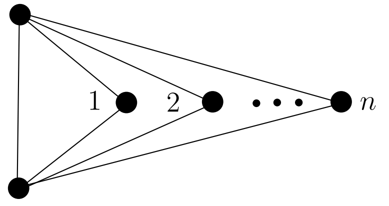

Fig. 1. $K'_{2,n}$.

(i) *G is bipartite;*
(ii) $G \cong K_4$*; or*
(iii) $G \cong K'_{2,n}$ *for some* $n \geq 1$.

There is a long history of generalizing results for graphs to binary matroids (see, for example, [3,7] or, more recently, [6, Section 15.4]). This paper continues this tradition by proving a generalization of Maffray's result. A circuit in a matroid is *even* if it has even cardinality; otherwise, it is *odd*. A *triangle* is a 3-element circuit. A binary matroid is *affine* if all of its circuits are even. Hence the cycle matroid, $M(G)$, of a graph G is affine if and only if G is bipartite. The following is the main result of the paper.

Theorem 1.2. *A connected simple binary matroid* M *has no odd circuits other than triangles if and only if*

(i) M *is affine;*
(ii) $M \cong M(K_4)$ *or* F_7*; or*
(iii) $M \cong M(K'_{2,n})$ *for some* $n \geq 1$.

The terminology used here will follow Oxley [6]. Binary affine matroids have several attractive characterizations. Indeed, Welsh [8] proved that the link between bipartite and Eulerian graphs via duality extends to binary matroids. His result is the equivalence of the first two parts of the next theorem (see, for example, [6, Theorem 9.4.1]). The equivalence of the first and third parts was proved independently by Brylawski [2] and Heron [4].

Theorem 1.3. *The following are equivalent for a binary matroid* M.

(i) M *is affine;*
(ii) M *is loopless and its simplification is isomorphic to a restriction of* $AG(r-1, 2)$ *for some* $r \geq 1$*;*
(iii) $E(M)$ *can be partitioned into cocircuits.*