

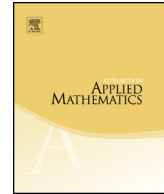


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Quadratic forms and congruences for ℓ -regular partitions modulo 3, 5 and 7

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ARTICLE INFO

Article history:

Received 2 March 2015

Received in revised form 29 June 2015

Accepted 29 June 2015

Available online 9 July 2015

MSC:

05A17

11P83

Keywords: ℓ -Regular partition

Quadratic form

Congruence

ABSTRACT

Let $b_\ell(n)$ be the number of ℓ -regular partitions of n . We show that the generating functions of $b_\ell(n)$ with $\ell = 3, 5, 6, 7$ and 10 are congruent to the products of two items of Ramanujan's theta functions $\psi(q)$, $f(-q)$ and $(q; q)_\infty^3$ modulo 3, 5 and 7. So we can express these generating functions as double summations in q . Based on the properties of binary quadratic forms, we obtain vanishing properties of the coefficients of these series. This leads to several infinite families of congruences for $b_\ell(n)$ modulo 3, 5 and 7.

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1. Introduction

An ℓ -regular partition of n is a partition of n such that none of its parts is divisible by ℓ . Denote the number of ℓ -regular partitions of n by $b_\ell(n)$. The arithmetic properties, the divisibility and the distribution of $b_\ell(n)$ have been widely studied in recent years.

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Alladi [1] studied the 2-adic behavior of $b_2(n)$ and $b_4(n)$ from a combinatorial point of view and obtained the divisibility results for small powers of 2. Lovejoy [12] proved the divisibility and distribution properties of $b_2(n)$ modulo primes $p \geq 5$ by using the theory of modular forms. Gordon and Ono [7] proved the divisibility properties of $b_\ell(n)$ modulo powers of the prime divisors of ℓ . Later, Ono and Penniston [14] studied the 2-adic behavior of $b_2(n)$. And Penniston [15] derived the behavior of p^a -regular partitions modulo p^j using the theory of modular forms.

The arithmetic properties of $b_\ell(n)$ modulo 2 have been widely investigated. Andrews, Hirschhorn and Sellers [2] derived some infinite families of congruences for $b_4(n)$ modulo 2. By applying the 2-dissection of the generating function of $b_5(n)$, Hirschhorn and Sellers [8] obtained many Ramanujan-type congruences for $b_5(n)$ modulo 2. Xia and Yao [16] established several infinite families of congruences for $b_9(n)$ modulo 2. Cui and Gu [4] derived congruences for $b_\ell(n)$ modulo 2 with $\ell = 2, 4, 5, 8, 13, 16$ by employing the p -dissection formulas of Ramanujan’s theta functions $\psi(q)$ and $f(-q)$.

As for the arithmetic properties of $b_\ell(n)$ modulo 3, Cui and Gu [5] and Keith [9] and Xia and Yao [17] studied respectively the congruences for $b_9(n)$ modulo 3. Lin and Wang [11] showed that 9-regular partitions and 3-cores satisfy the same congruences modulo 3 and further generalized Keith’s conjecture and derived a stronger result. Furcy and Penniston [6] obtained congruences for $b_\ell(n)$ modulo 3 with $\ell = 4, 7, 13, 19, 25, 34, 37, 43, 49$ by using the theory of modular forms.

Notice that all the above congruences for $b_\ell(n)$ were proven by using modular forms or elementary q -series manipulations. In this paper, we take a different approach which is based on the properties of binary quadratic forms. Lovejoy and Osburn [13] generalized the congruences modulo 3 for four types of partitions by employing the representations of numbers as certain quadratic forms. Employing the arithmetic properties of quadratic forms, Kim [10] proved that the number of overpartition pairs of n is almost always divisible by 2^8 .

We derive infinite families of congruence relations for ℓ -regular partitions with $\ell = 3, 5, 6, 7, 10$ modulo 3, 5 and 7 by establishing a general method (see Proposition 2.1). Our method is based on a bivariate extension of Cui and Gu’s approach [4].

Notice that the generating function of $b_\ell(n)$ is given by

$$B_\ell(q) = \sum_{n=0}^{\infty} b_\ell(n)q^n = \frac{(q^\ell; q^\ell)_\infty}{(q; q)_\infty},$$

where

$$(q; q)_\infty = \prod_{i=1}^{\infty} (1 - q^i)$$

is the standard notation in q -series. Let $\psi(q)$ and $f(-q)$ be Ramanujan’s theta functions given by

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