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On matroid minors that guarantee their duals as minors

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A natural question for matroids follows: Which matroids N guarantee that, when present as minors in a larger matroid M, their duals are present as minors? The main results of this paper focus on this question with the additional constraint that N and M are 3-connected. We also solve the problem with other connectivity and representability constraints.

1. Introduction

For matroids M and N, in order for M to have both N and N^* as minors, we must have $\min\{r(M), r^*(M)\} \geq \max\{r(N), r^*(N)\}$. Subject to these *obligatory rank constraints*, the goal of this paper is to find all matroids N such that M has an N-minor if and only if M has an N^*-minor. The main results of the paper, stated below, consider the case where N and M are 3-connected.

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Theorem 1.1. *Let N be a 3-connected matroid that is not self-dual, and let M be a 3-connected matroid for which*

$$\min\{r(M), r^*(M)\} \geq \max\{r(N), r^*(N)\}.$$

The following are equivalent:

(i) *M has an N-minor if and only if M has an N^*-minor.*
(ii) *The matroid N has fewer than four elements, or is $U_{2,5}$ or $U_{3,5}$.*

Theorem 1.2. *Let N be a 3-connected binary matroid that is not self-dual, and let M be a 3-connected binary matroid for which*

$$\min\{r(M), r^*(M)\} \geq \max\{r(N), r^*(N)\}.$$

The following are equivalent:

(i) *M has an N-minor if and only if M has an N^*-minor.*
(ii) *The matroid N has fewer than four elements, or is F_7 or F_7^*.*

We also consider this question with other connectivity and representability constraints. In Section 2, we introduce some preliminaries and discuss the problem with no restrictions on connectivity or representability. We also discuss the problem for 2-connected matroids in Section 2. Section 3 is devoted to proving Theorem 1.1 and the analogous result for matroids representable over infinite fields. In Section 4, we solve the problem for 3-connected, $GF(q)$-representable matroids and prove Theorem 1.2. Except where otherwise noted, the notation and terminology throughout will follow [4].

2. Preliminaries

We begin with a few observations. First, consider a self-dual matroid N. Clearly, every matroid M that contains N as a minor also contains N^* as a minor. Therefore, we restrict our interest to matroids that are not self-dual. Now let N be an arbitrary matroid. We define $\delta(N) = r(N) - r^*(N)$. If $\delta(N) = 0$, then it is easy to see that either N is self-dual, or there is a matroid M that satisfies the obligatory rank constraints and has N but not N^* as a minor. Thus, without loss of generality, we may assume that $r(N) > r^*(N)$, and so $\delta(N) > 0$.

We now describe a strategy that will be used in many proofs throughout this paper. Beginning with a matroid N, we construct a matroid M by adjoining $\delta(N)$ elements to N in such a way that we do not increase the rank of N. Then we increase the corank of N by $\delta(N)$. This matroid M satisfies the obligatory rank constraints. Note that, when we employ this strategy, we must contract $\delta(N)$ elements from M if we hope to obtain