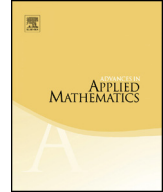




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A new companion to Capparelli's identities



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ABSTRACT

We discuss a new companion to Capparelli's identities. Capparelli's identities for $m = 1, 2$ state that the number of partitions of n into distinct parts not congruent to m , $-m$ modulo 6 is equal to the number of partitions of n into distinct parts not equal to m , where the difference between parts is greater than or equal to 4, unless consecutive parts are either both consecutive multiples of 3 or add up to a multiple of 6. In this paper we show that the set of partitions of n into distinct parts where the odd-indexed parts are not congruent to m modulo 3, the even-indexed parts are not congruent to $3 - m$ modulo 3, and $3l + 1$ and $3l + 2$ do not appear together as consecutive parts for any integer l has the same number of elements as the above mentioned Capparelli's partitions of n . In this study we also extend the work of Alladi, Andrews and Gordon by providing a complete set of generating functions for the refined Capparelli partitions, and conjecture some combinatorial inequalities.

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1. Introduction and notation

A partition π is a finite, non-increasing sequence of positive integers $(\pi_1, \pi_2, \dots, \pi_k)$. The π_i are called parts of the partition π , and $\pi_1 + \pi_2 + \dots + \pi_k$ is called the norm of π . We call π “a partition of n ” if the norm of π is n . Conventionally, we define empty sequence as the only partition of 0. Throughout this paper we assume that a and q are complex numbers where $|q| < 1$, L is a positive integer, and $m \in \{1, 2\}$. We use the standard notations as in [3] and [10]:

$$(a)_L := (a; q)_L = \prod_{n=0}^{L-1} (1 - aq^n),$$

$$(a_1, a_2, \dots, a_k; q)_L = (a_1; q)_L (a_2; q)_L \dots (a_k; q)_L,$$

$$(a; q)_\infty := \prod_{n=0}^{\infty} (1 - aq^n).$$

We define the q -binomial and q -trinomial coefficients, respectively, as

$$\begin{bmatrix} k \\ n \end{bmatrix}_q := \begin{cases} \frac{(q)_k}{(q)_n (q)_{k-n}} & \text{for } k \geq n \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\begin{bmatrix} k \\ n, r \end{bmatrix}_q := \begin{bmatrix} k \\ n \end{bmatrix}_q \begin{bmatrix} k-n \\ r \end{bmatrix}_q = \begin{cases} \frac{(q)_k}{(q)_n (q)_r (q)_{k-n-r}} & \text{for } k \geq n+r \geq n \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Let $C_m(n)$ be the number of partitions of n into distinct parts where no part is congruent to $\pm m$ modulo 6. Define $D_m(n)$ to be the number of partitions of n into parts, not equal to m , where the minimal difference between consecutive parts is 2, in fact, the difference between consecutive parts is greater than or equal to 4 unless these parts either are both consecutive multiples of 3 (yielding a difference of 3) or add up to a multiple of 6 (possibly giving a difference of 2).

In 1988, S. Capparelli stated two conjectures for C_m and D_m in his thesis [7]. The first one was later proven by G.E. Andrews [2] in 1992 during the Centenary Conference in Honor of Hans Rademacher. Two years later Lie’s theoretic proof was supplied by Tamba and Xie [12] and by Capparelli [8]. The first of Capparelli’s conjectures was stated and proven in the form of [Theorem 1.1](#).

Theorem 1.1 (Andrews 1992). *For any non-negative integer n ,*

$$C_1(n) = D_1(n).$$

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