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Ramanujan's Master Theorem applied to the evaluation of Feynman diagrams

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A B S T R A C T

Ramanujan's Master Theorem is a technique developed by S. Ramanujan to evaluate a class of definite integrals. This technique is used here to produce the values of integrals associated with Feynman diagrams.

1. Introduction

Precise experimental measurements in high energy physics require, in its theoretical counterpart, the development of new techniques for the evaluation of analytic objects

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associated with the corresponding Feynman diagrams. These techniques have lately emphasized the automatization of calculations of multiscale, multiloop diagrams.

Modern numerical methods for the evaluation of Feynman diagrams benefit from analytical techniques employed as preliminary work to detect the presence of divergences. Recent advances include a method based on the Bernstein–Tkachov theorem for the corrections of one and two loop diagrams and methods based on sector-decompositions. New analytic methods to reduce Feynman diagrams to a small number of scalar integrals include integration by parts, the use of Lorenz invariance and other symmetries, Mellin–Barnes transforms and differential equations. The reader is referred to [11] for a description of these and other methods for the evaluation of Feynman diagrams and to [12,13] for readable introductions to the topic.

This paper contains examples of an alternative method for the evaluation of some Feynman diagrams. It is based on the classical Ramanujan's Master Theorem, one of his favorite techniques to evaluate definite integrals. The theoretical aspects of this method are presented in [9] and a collection of examples and some justification of the algorithm is given in [1,4,6]. This technique has also been used in [7] for the evaluation of some multidimensional integrals obtained by the Schwinger parametrization of Feynman diagrams.

The goal of the present work is to illustrate the flexibility of the method by evaluating integrals associated with two and three loop diagrams. Naturally the method works for a large variety of definite integrals and the first example illustrates this by computing the Mellin transform of a Bessel function. The automatization of this process began in [10] and progress is reported in [5].

2. Advantage of Ramanujan's Master Theorem in the evaluation of Feynman diagrams

The integral representation associated with a Feynman diagram associated with a scalar field comes directly from the directed graph G of the diagram. Assume the graph has N propagators (or *internal lines*), L loops (attached to the flux of independent internal momenta) $\underline{q} = (q_1, ..., q_L)$, E external independent momenta (*external lines*) $\underline{p} = (p_1, ..., p_E)$ and each propagator is characterized by its mass $(m_1, ..., m_N)$. The integral representing the diagram in D-dimensions is

$$G = \int \frac{d^D q_1}{i\pi^{D/2}} \cdots \frac{d^D q_L}{i\pi^{D/2}} \frac{1}{(B_1^2 - m_1^2 + i0)^{\nu_1}} \cdots \frac{1}{(B_N^2 - m_N^2 + i0)^{\nu_N}}. \tag{2.1}$$

In this expression, the symbol B_j represents the momentum of the j-th propagator, given as a linear combination of the momenta, both external $\{\underline{p}\}$ and internal $\{\underline{q}\}$. The propagators also have a sequence of arbitrary powers $\underline{\nu} = \{\nu_1, ..., \nu_N\}$, one per propagator.

The evaluation of this integral begins with the so-called *parametrization*. This simple corresponds to a formulation of the problem as an integral in N-dimensional space. The