

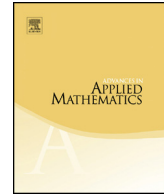


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The sorting index on colored permutations and even-signed permutations [☆]



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ABSTRACT

We define a new statistic sor on the set of colored permutations $G_{r,n}$ and prove that it has the same distribution as the length function. For the set of restricted colored permutations corresponding to the arrangements of n non-attacking rooks on a fixed Ferrers shape we show that the following two sequences of set-valued statistics are joint equidistributed: $(\ell, \text{Rmil}^0, \text{Rmil}^1, \dots, \text{Rmil}^{r-1}, \text{Lmil}^0, \text{Lmil}^1, \dots, \text{Lmil}^{r-1}, \text{Lmal}^0, \text{Lmal}^1, \dots, \text{Lmal}^{r-1}, \text{Lmap}^0, \text{Lmap}^1, \dots, \text{Lmap}^{r-1})$ and $(\text{sor}, \text{Cyc}^0, \text{Cyc}^{r-1}, \dots, \text{Cyc}^1, \text{Lmic}^0, \text{Lmic}^{r-1}, \dots, \text{Lmic}^1, \text{Lmal}^0, \text{Lmal}^1, \dots, \text{Lmal}^{r-1}, \text{Lmap}^0, \text{Lmap}^1, \dots, \text{Lmap}^{r-1})$. Analogous results are also obtained for Coxeter group of type D . Our work generalizes recent results of Petersen, Chen–Gong–Guo and Poznanović.

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1. Introduction

1.1. Mahonian and Stirling statistics

Let \mathfrak{S}_n be the group of permutations on n letters $[n] := \{1, 2, \dots, n\}$. A pair (σ_i, σ_j) is called an *inversion* in a permutation $\sigma = \sigma_1 \cdots \sigma_n \in \mathfrak{S}_n$ if $i > j$ and $\sigma_i < \sigma_j$. Denote by $\text{inv}(\sigma)$ the number of inversions in σ . The distribution of inv over \mathfrak{S}_n was first found by Rodriguez [9] to be

$$\sum_{\sigma \in \mathfrak{S}_n} q^{\text{inv}(\sigma)} = \prod_{i=1}^n [i]_q, \quad (1.1)$$

where $[i]_q := 1 + q + \cdots + q^{i-1}$.

In a Coxeter group, the *length* $\ell(\sigma)$ of a group element σ is the minimal number of generators needed to express σ . It is well known [2, Chapter 8] that \mathfrak{S}_n is the Coxeter group of type A, where the generators are the adjacent transpositions and $\ell(\sigma) = \text{inv}(\sigma)$. A permutation statistic is called *Mahonian* if it is equidistributed with inv over \mathfrak{S}_n . Similarly in a Coxeter group a statistic is called *Mahonian* if it is equidistributed with the length function ℓ .

The number of cycles cyc is another important statistic, whose distribution over \mathfrak{S}_n is [10, Proposition 1.3.4]

$$\sum_{\sigma \in \mathfrak{S}_n} t^{\text{cyc}(\sigma)} = \prod_{i=1}^n (t + i - 1). \quad (1.2)$$

As the coefficients of this polynomial are the (unsigned) Stirling numbers of the first kind, a permutation statistic over \mathfrak{S}_n is called *Stirling* if it is equidistributed with cyc .

The *reflection length* $\ell'(\sigma)$ of σ in a Coxeter group is the minimal number of reflections (i.e., elements conjugate to generators) needed to express σ . In type A, the reflections are the transpositions and one has

$$\text{cyc}(\sigma) = n - \ell'(\sigma). \quad (1.3)$$

1.2. Sorting index

Petersen [7] defined the *sorting index* sor over \mathfrak{S}_n and proved it is Mahonian. One can uniquely decompose $\sigma \in \mathfrak{S}_n$ into a product of transpositions

$$\sigma = (i_1 j_1)(i_2 j_2) \cdots (i_k j_k)$$

with $j_1 < j_2 < \cdots < j_k$ and $i_1 < j_1, i_2 < j_2, \dots, i_k < j_k$. Then the sorting index of σ is

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