Advances in Applied Mathematics 65 (2015) 38–64

Contents lists available at ScienceDirect

Advances in Applied Mathematics

www.elsevier.com/locate/yaama

Random unfriendly seating arrangement in a dining table

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A R T I C L E I N F O

Article history:
Received 25 June 2014
Received in revised form 15 November 2014
Accepted 20 January 2015
Available online 2 March 2015

MSC:
primary 60C05
secondary 60D05, 60F05, 60E15

Keywords:
Riccati equation
Jamming density
Quasi-power theorem
Central and local limit theorem
Stochastic dominance

A B S T R A C T

A detailed study is made of the number of occupied seats in an unfriendly seating scheme with two rows of seats. An unusual identity is derived for the probability generating function, which is itself an asymptotic expansion. The identity implies particularly a local limit theorem with optimal convergence rate. Our approach relies on the resolution of Riccati equations. We also clarify some simple yet delicate stochastic dominance relations.

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[1] Partially supported by a research grant from Ministry of Science and Technology under the Grant MOST 103-2118-M-001-004-MY3, Taiwan.

http://dx.doi.org/10.1016/j.aam.2015.01.002
0196-8858/

1. Introduction

Freedman and Shepp formulated the "unfriendly seating arrangement problem" in 1962 [16, Problem 62–3]:

There are n seats in a row at a luncheonette and people sit down one at a time at random. They are unfriendly and so never sit next to one another (no moving over). What is the expected number of persons to sit down?

Let Z_n denote the number of persons sitting down when no further customers can sit properly without breaking the restriction of unfriendliness. Solutions with different degree of precision or generality were later proposed by many. In particular, Friedman and Rothman [17] proved that

$$\begin{aligned}\mathbb{E}(Z_n) &= \sum_{0\leqslant k<n} (n-k)\frac{(-2)^k}{(k+1)!} \\ &= \tfrac{1}{2}\left(1-e^{-2}\right)(n+3) - 1 + O\left(\frac{2^n}{(n+2)!}\right),\end{aligned}$$

for large n. The *factorial* error term here seems characteristic of sequential models of a similar nature; see, for example, (1), (14) and (16) below and [6]. We will provide a general framework for characterizing such small errors; see Proposition 1 below. In addition, Friedman and Rothman [17] extended the "degree of unfriendliness" to any integer $b \geqslant 1$, where any two people have to sit with at least b unoccupied seats between them. This extension was mentioned to be related to Rényi's Parking Problem and to a discrete parking problem studied by MacKenzie (see [22]) in which cars of the same length $\ell \geqslant 2$ are parked uniformly at random along the curb with n unit parking spaces. Indeed, the latter problem with $\ell = 2$ found its origin in Flory's 1939 pioneering paper [15] in polymer chemistry, and was later expanded into generic stochastic models under the name "random sequential adsorption"; see [7] for a comprehensive survey and [2,8,25,26] a more recent account.

Due to the simplicity and the usefulness of the model, the same discrete parking problem was also studied independently under different guises in applied probability and related areas. Page [23] studied a random pairing model in which n isolated points are paired randomly by adjacency until only singletons remain. This model is identical to Flory's monomer-dimer model [15] (or the discrete parking problem [22] where each car requires 2-unit parking space). The same model was also encountered in a few diverse modeling contexts. Let ζ_n denote the resulting number of pairs when no more adjacent pair can be formed. Then it is easy to see that

$$Z_n \equiv \tfrac{1}{2}\zeta_{n+1} \qquad (n \geqslant 0).$$