

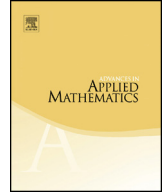


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Pattern avoidance in ordered set partitions and words [☆]



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ABSTRACT

We consider the enumeration of ordered set partitions avoiding a permutation pattern, as introduced by Godbole, Goyt, Herdan and Pudwell. Let $op_{n,k}(p)$ be the number of ordered partitions of $\{1, 2, \dots, n\}$ into k blocks that avoid a permutation pattern p . We establish an explicit identity between the number $op_{n,k}(p)$ and the number of words avoiding the inverse of p . This identity allows us to easily translate results on pattern-avoiding words obtained in earlier works into equivalent results on pattern-avoiding ordered set partitions. In particular, (a) we determine the asymptotic growth rate of the sequence $(op_{n,k}(p))_{n \geq 1}$ for every positive k and every permutation pattern p , (b) we partially confirm a conjecture of Godbole et al. concerning the variation of the sequences $(op_{n,k}(p))_{1 \leq k \leq n}$, (c) we undertake a detailed study of the number of ordered set partitions avoiding a permutation pattern of length 3. By the way, we observe that the number of words on k letters of length n avoiding a permutation pattern of length 3 can be written as a single sum by simplifying a formula of Burstein.

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1. Introduction

1.1. This paper is concerned with the enumeration of pattern-avoiding ordered set partitions. The subject, which can be seen as a generalization of the study of pattern-avoiding permutations, was initiated by Godbole, Goyt, Herdan and Pudwell in [7]. Recall that an ordered partition of a set S is a sequence of nonempty and mutually disjoint subsets, called blocks, whose union is S . When S is a subset of integers, it is usual to separate the blocks by a slash and to arrange the elements within each block in increasing order. We let $\mathcal{OP}_{n,k}$ denote the set of ordered partitions of $[n] := \{1, 2, \dots, n\}$ into k blocks. For instance, $27/3/148/56$ and $3/27/148/56$ are two (distinct) members of $\mathcal{OP}_{8,4}$.

Godbole et al. [7] considered the following notion of pattern containment: an ordered partition $\pi = B_1/B_2/\dots/B_k$ in $\mathcal{OP}_{n,k}$ is said to contain a permutation $p = p_1 p_2 \dots p_m$ in \mathfrak{S}_m , the symmetric group of the set $[m]$, as a pattern if there exist integers i_1, i_2, \dots, i_m , with $1 \leq i_1 < i_2 < \dots < i_m \leq k$, and there exists $b_j \in B_{i_j}$ such that $b_1 b_2 \dots b_m$ is order isomorphic to p . Otherwise we say that π avoids p . For example, $27/3/148/56$ contains the pattern $p = 213$, as evidenced (for instance) by $b_1 = 2 \in B_1$, $b_2 = 1 \in B_3$ and $b_3 = 5 \in B_4$. The number of members of $\mathcal{OP}_{n,k}$ that avoid the pattern p will be denoted by $\text{op}_{n,k}(p)$. The special case $k = n$ is of particular interest since we clearly have

$$\text{op}_{n,n}(p) = s_n(p), \tag{1.1}$$

where $s_n(p)$ stands for the number of permutations in \mathfrak{S}_n that avoid the pattern p in the usual sense (see e.g. [2, Chapter 4]).

Finding a closed formula for $\text{op}_{n,k}(p)$ is in general a hopeless task. Obviously, we have $\text{op}_{n,k}(1) = 0$ for all $n \geq k \geq 1$. It is also easy to show [7] that

$$\text{op}_{n,k}(12) = \text{op}_{n,k}(21) = \binom{n-1}{k-1} \quad (n \geq k \geq 1). \tag{1.2}$$

The first nontrivial case is that of patterns of length three. Godbole et al. proved that $\text{op}_{n,k}(p)$, for any $n, k \geq 1$, is the same for all $p \in \mathfrak{S}_3$, i.e. $\text{op}_{n,k}(p) = \text{op}_{n,k}(321)$ for all $p \in \mathfrak{S}_3$. By (1.1), this nicely generalizes Knuth’s observation (see e.g. [2, Chapter 4]) that the number of permutations of $[n]$ that avoid the pattern p is the same for all $p \in \mathfrak{S}_3$. Table 1 lists the first few values of $\text{op}_{n,k}(321)$. That the first values of the diagonal $(\text{op}_{n,n}(321))_{n \geq 1}$ coincide with those of the Catalan sequence comes as no surprise because of Knuth’s well-known result (see e.g. [2, Chapter 4]) that $s_n(p)$, for all $p \in \mathfrak{S}_3$, is the n -th Catalan number $C_n = \frac{1}{n+1} \binom{2n}{n}$.

The study of the array $[\text{op}_{n,k}(321)]_{n \geq k \geq 1}$, which has also been considered in [7,5], is one of the central themes in this paper with which Section 3 is wholly concerned. We will also derive interesting results for general pattern p . To this end, we first need to state

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