Advances in Applied Mathematics 64 (2015) 124–200

Contents lists available at ScienceDirect

Advances in Applied Mathematics

www.elsevier.com/locate/yaama

An algorithm for deciding the finiteness of the number of simple permutations in permutation classes ☆

Frédérique Bassino [a], Mathilde Bouvel [b,c,*], Adeline Pierrot [d,e,f], Dominique Rossin [g]

[a] Université Paris 13, Sorbonne Paris Cité, LIPN, CNRS UMR 7030, F-93430, Villetaneuse, France
[b] LaBRI UMR 5800, Univ. Bordeaux and CNRS, 351, cours de la Libération, 33405 Talence cedex, France
[c] Institut für Mathematik, Uni. Zürich, Zurich, Switzerland
[d] LIAFA UMR 7089, Univ. Paris Diderot and CNRS, Case 7014, 75205 Paris cedex 13, France
[e] Institut für Diskrete Mathematik und Geometrie, TU Wien, Vienna, Austria
[f] LRI UMR 8623, Univ. Paris Sud and CNRS, 91405 Orsay Cedex, France
[g] LIX UMR 7161, Ecole Polytechnique and CNRS, 91128 Palaiseau, France

ARTICLE INFO

Article history:
Received 8 August 2013
Received in revised form 8 December 2014
Accepted 8 December 2014
Available online 6 January 2015

MSC:
05A05
68R05
05-04

Keywords:
Permutation class
Finite basis
Simple permutation

ABSTRACT

In this article, we describe an algorithm to determine whether a permutation class $\mathcal{C}$ given by a finite basis B of excluded patterns contains a finite number of simple permutations. This is a continuation of the work initiated in [11], and shares several aspects with it. Like in this article, the main difficulty is to decide whether $\mathcal{C}$ contains a finite number of proper pin-permutations, and this decision problem is solved using automata theory. Moreover, we use an encoding of proper pin-permutations by words over a finite alphabet, introduced by Brignall et al. However, unlike in their article, our construction of automata is fully algorithmic and efficient. It is based on the study of pin-permutations in [6]. The complexity of the overall algorithm is $\mathcal{O}(n \log n + s^{2k})$ where n denotes the sum of the sizes of permutations in the basis B,

☆ This work was completed with the support of the ANR (project ANR BLAN-0204_07 MAGNUM).
* Corresponding author at: Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zurich, Switzerland.
E-mail address: mathilde.bouvel@math.uzh.ch (M. Bouvel).

http://dx.doi.org/10.1016/j.aam.2014.12.001
0196-8858/

Algorithm
Automaton
Pin-permutation

s is the maximal size of a pin-permutation in B and k is the number of pin-permutations in B.

1. Introduction

Since the definition of the pattern relation among permutations by Knuth in the 1970s [17], the study of permutation patterns and permutation classes in combinatorics has been a quickly growing research field, and is now well-established. Most of the research done in this domain concerns *enumeration* questions on permutation classes. Another line of research on permutation classes has been emerging for about a decade: it is interested in properties or results that are less precise but apply to *families* of permutation classes. Examples of such general results may regard enumeration of permutation classes that fall into general frameworks, properties of the corresponding generating functions, growth rates of permutation classes, order-theoretic properties of permutation classes .... This second point of view is not purely combinatorial but instead is intimately linked with algorithms. Indeed, when stating general structural results on families of permutation classes, it is natural to associate to an existential theorem an algorithm that *tests* whether a class given in input falls into the family of classes covered by the theorem, and in this case to *compute* the result whose existence is assessed by the theorem.

Certainly the best illustration of this paradigm that can be found in the literature is the result of Albert and Atkinson [3], stating that every permutation class containing a finite number of simple permutations has a finite basis and an algebraic generating function, and its developments by Brignall et al. in [9–11]. A possible interpretation of this result is that the simple permutations that are contained in a class somehow determine how structured the class is. Indeed, the algebraicity of the generating function is an echo of a deep structure of the class that appears in the proof of the theorem of [3]: the permutations of the class (or rather their decomposition trees) can be described by a context-free grammar. In this theorem, as well as in other results obtained in this field, it appears that *simple permutations* play a crucial role. They can be seen as encapsulating most of the difficulties in the study of permutation classes considered in their generality, both in algorithms and combinatorics.

Our work is about these general results that can be obtained for large families of permutation classes, and is resolutely turned towards algorithmic considerations. It takes its root in the theorem of Albert and Atkinson that we already mentioned, and follows its developments in [11] and [6].

In [11], Brignall, Ruškuc and Vatter provide a criterion on a finite basis B for deciding whether a permutation class $\mathcal{C} = Av(B)$ contains a finite number of simple permutations. We have seen from [3] that this is a sufficient condition for the class to be well-structured. To this criterion, [11] associates a decision procedure testing from a finite basis B whether