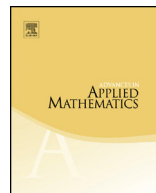




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# The structure of 4-flowers of vertically 4-connected matroids <sup>☆</sup>



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## ABSTRACT

Aikin and Oxley (2012) [2] studied the structure of 4-flowers in 4-connected matroids. In the paper we consider 4-flowers in vertically 4-connected matroids. There is a natural relation of equivalence on such 4-flowers. We characterize the structure that arises when 4-flowers are equivalent under the relation.

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## 1. Introduction

In the paper we study 4-flowers in vertically 4-connected matroids. That extends work of Aikin and Oxley [2]. In particular, we describe the structure that arises when 4-flowers are equivalent.

Before stating our main theorem, we need some technical preliminaries. The matroid terminology used here mainly follows Oxley [5]. Let  $E$  be the ground set of a matroid  $M$ .

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The *local connectivity* of two subsets  $X$  and  $Y$  of  $E$ , denoted by  $\square(X, Y)$ , is defined by  $\square(X, Y) = r(X) + r(Y) - r(X \cup Y)$ . If  $(X, Y)$  is a partition of  $E$ , then we say  $\lambda(X) = \square(X, E - X)$ . Let  $k \geq 1$  be an integer. If  $\lambda(X) \leq k - 1$ , then we say that  $X$  or  $(X, E - X)$  is *k-separating*. If  $\lambda(X) = k - 1$ , then  $X$  or  $(X, E - X)$  is *exactly k-separating*. If  $1 \leq k \leq \ell$ , then we set  $[\ell] = \{1, 2, \dots, \ell\}$  and  $[k, \ell] = \{k, k + 1, \dots, \ell\}$ . An ordered partition  $\Phi = (P_1, P_2, \dots, P_n)$  of the ground set  $E$  of a matroid  $M$  is a *k-flower* with petals  $P_1, P_2, \dots, P_n$  if each  $P_i$  is exactly *k-separating* and, when  $n \geq 3$ , each  $P_i \cup P_{i+1}$  is exactly *k-separating*. When  $M$  is 3-connected, a 3-flower is what is defined in [6] as a flower. Assume that  $I$  is a proper non-empty subset of  $[n]$ . Then  $\Phi$  is a *k-anemone* if  $P_I = \bigcup_{i \in I} P_i$  is exactly *k-separating* for all such  $I$ ; and  $\Phi$  is a *k-daisy* if  $P_I$  is exactly *k-separating* for precisely those subsets  $I$  whose numbers form a consecutive set in the cyclic order  $(1, 2, \dots, n)$ . Aikin and Oxley [1] generalized a result of [6] by showing that every non-trivial *k-flower* is either a *k-anemone* or a *k-daisy*. For an arbitrary petal  $P_i$  of  $\Phi$ , when  $\Phi$  is a *k-anemone*, any petal is adjacent to  $P_i$  *up to labels*; and when  $\Phi$  is a *k-daisy*, a petal is adjacent to  $P_i$  *up to labels* if it is adjacent to  $P_i$  in the cyclic order  $(P_1, P_2, \dots, P_n)$ .

Let  $X$  be an exactly 3-separating set of a 3-connected matroid  $M$ . The *full closure* of  $X$ , denoted by  $\text{fcl}(X)$ , is the minimum set containing  $X$  such that no element  $e \in E - \text{fcl}(X)$  satisfies  $\lambda(\text{fcl}(X) \cup \{e\}) \leq 2$ . An element  $e \in P_i$  is *loose* if there is some petal  $P_j$  adjacent to  $P_i$  up to labels satisfying  $e \in \text{fcl}(P_j)$ . Petal  $P_i$  is *loose* if every element of it is loose, otherwise it is *tight*. And  $\Phi$  is *tight* if every petal of it is tight.

In the rest of this section, all 4-flowers are in a vertically 4-connected matroid with rank at least seven. Let  $\Phi = (P_1, \dots, P_n)$  be a 4-flower. By [1, Lemma 3.4], there is an integer, denoted by  $c(\Phi)$ , that represents the local connectivity between any pair of consecutive petals. If  $n \geq 5$ , then by [1, Lemma 3.4], there is an integer, denoted by  $d(\Phi)$ , that represents the local connectivity between any pair of nonconsecutive petals. If  $n = 4$ , then  $\square(P_1, P_3)$  may be not equal to  $\square(P_2, P_4)$ . In this paper, we will completely characterize the structure of 4-flowers with equal local connectivities for any two nonconsecutive petals. By [1, Theorems 1.3 and 1.4], there are six types for such 4-flowers, that is, 4-anemones with  $c \in \{0, 1, 2, 3\}$  and 4-daisies of type  $(2, 1)$  or  $(1, 0)$ , where a 4-flower of type  $(c, d)$  means a 4-flower with  $c(\Phi) = c$  and  $d(\Phi) = d$ .

For any tight 3-flower  $\Phi$  with at least four petals of a 3-connected matroid, Oxley, Semple and Whittle [6] completely characterized its structure, which enables us to find all equivalent 3-flowers of  $\Phi$ . On the other hand, Chen and Xiang [3] recently proved that via an operation “reducing”, every 3-connected representable matroid  $M$  with at least nine elements can be decomposed into a set of sequentially 4-connected matroids and three special classes of matroids. Moreover, via an operation similar to “reducing”, sequentially 4-connected matroids can be decomposed into vertically 4-connected matroids and a special class of matroid. Thus, it is interesting to know more about the structure of 4-flowers of vertically 4-connected matroids. To characterize the structure of tight 4-flowers with at least four petals of a vertically 4-connected matroid with rank at least seven, first we must generalize the definition of full-closure to allow moving rank-2 sets like [2, 4]. Here is an example to illustrate this, which is similar to the one given in [2].

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