

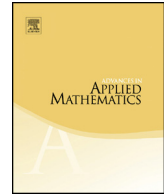


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Sign imbalances of snakes and valley-signed permutations [☆]



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ABSTRACT

One of the combinatorial structures counted by the Springer numbers is the set of snakes, which in type A_n is the set of the alternating permutations and in type B_n (or D_n) is the set of certain signed permutations. The set of valley-signed permutations, defined by Josuat-Vergès, Novelli and Thibon, is another structure counted by the Springer numbers of type B_n (or D_n). In this paper we determine the sign imbalances of these sets of snakes and valley-signed permutations under various inversion statistics inv_w , inv_o , inv_s , inv_B , and inv_D .

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1. Introduction

1.1. Springer numbers and snakes

The Springer numbers were introduced by Springer [14] in the study of the Coxeter groups. Let (W, S) be an irreducible Coxeter system and ℓ be the length function. We say $D(w) := \{s \in S : \ell(ws) < \ell(w)\}$ is the descent set of $w \in W$. The Springer number of (W, S) is defined by

$$K(W) := \max_{J \subseteq S} |\mathcal{D}_J|, \tag{1.1}$$

where $\mathcal{D}_J = \{w \in W : D(w) = J\}$. The sequences of Springer numbers of types A_n, B_n and D_n ($n \geq 0$) are respectively

- type A_n : 1, 1, 1, 2, 5, 16, 61, 272, 1385, 7936, 50 521, ...
- type B_n : 1, 1, 3, 11, 57, 361, 2763, 24 611, 250 737, 2 873 041, 36 581 523, ...
- type D_n : 1, 1, 1, 5, 23, 151, 1141, 10 205, 103 823, 1 190 191, 15 151 981, ...

Let \mathfrak{S}_n be the set of permutations of $[n] := \{1, 2, \dots, n\}$. A permutation $\pi = \pi_1 \pi_2 \dots \pi_n \in \mathfrak{S}_n$ is *alternating* if $\pi_1 > \pi_2 < \pi_3 > \pi_4 < \dots$. Let \mathbf{E}_n be the set of alternating permutations of $[n]$ and it is well known [17] that $|\mathbf{E}_n|$ is the Euler number.

Let $[\pm n] := \{-n, \dots, -1\} \cup \{1, \dots, n\}$. A *signed permutation* $\pi = \pi_{-n} \dots \pi_{-1} \pi_1 \dots \pi_n$ (in its *one-line notation*) is a bijection of $[\pm n]$ to itself such that $\pi(-i) = -\pi(i)$ for all $i \in [\pm n]$. For simplicity we denote $-i$ by \bar{i} . The *window notation* of π is $\pi = [\pi_1 \pi_2 \dots \pi_n]$. For example, the signed permutation $\pi = 1 \bar{4} \bar{5} 3 \bar{2} 2 \bar{3} 5 4 \bar{1}$ has the window notation $[2 \bar{3} 5 4 \bar{1}]$. Throughout this paper, if there is no danger of confusion, we simply write $2 \bar{3} 5 4 \bar{1}$ instead of $[2 \bar{3} 5 4 \bar{1}]$.

Let \mathfrak{S}_n^\pm denote the set of signed permutations of $[\pm n]$ and $\mathfrak{S}_n^D \subseteq \mathfrak{S}_n^\pm$ denote the set of signed permutations π with an even number of negatives among π_1, \dots, π_n . It is known [2] that $\mathfrak{S}_n, \mathfrak{S}_n^\pm$ and \mathfrak{S}_n^D are respectively combinatorial descriptions of the Coxeter groups of types A_n, B_n and D_n . A variety of *signed alternating permutations* are given as follows.

- $\mathbf{S}_n := \{\pi \in \mathfrak{S}_n^\pm : \pi_1 > \pi_2 < \pi_3 > \pi_4 < \dots\}$,
- $\mathbf{S}_n^0 := \{\pi \in \mathfrak{S}_n^\pm : \pi_1 > 0 \text{ and } \pi_1 > \pi_2 < \pi_3 > \pi_4 < \dots\}$,
- $\mathbf{D}_n := \{\pi \in \mathfrak{S}_n^D : \pi_1 + \pi_2 < 0 \text{ and } \pi_1 > \pi_2 < \pi_3 > \pi_4 < \dots\}$.

Arnol'd [1] proved that $\mathbf{E}_n, \mathbf{S}_n^0$ and \mathbf{D}_n are equal to some maximum sets \mathcal{D}_J as introduced in Eq. (1.1) for W the Coxeter groups of types A_n, B_n and D_n , respectively. Therefore, the cardinalities of $\mathbf{E}_n, \mathbf{S}_n^0$ and \mathbf{D}_n are the Springer numbers of types A_n, B_n and D_n , respectively. Arnol'd coined the term *snakes* to describe the (signed) alternating

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