

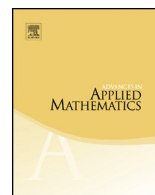


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Functions of the second kind for classical polynomials [☆]

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ABSTRACT

We give new derivations of properties of the functions of the second kind of the Jacobi, little and big q -Jacobi polynomials, and the symmetric Al-Salam–Chihara polynomials for $q > 1$. We also study the Askey–Wilson functions and the Wilson functions of second kind. An integration by parts formula is derived for the Wilson operator in appropriate Hilbert space. In case of undetermined moment problem, we prove that the function of second kind depends on the measure.

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1. Introduction

Given a sequence of polynomials $\{P_n(x)\}$ orthogonal with respect to a weight function $w(x)$ on (a, b) , $-\infty \leq a < b \leq +\infty$, the function of the second kind $Q_n(z)$ is defined as

$$Q_n(z) = \frac{1}{w(z)} \int_a^b \frac{P_n(y)}{z-y} w(y) dy, \quad z \in \mathbb{C} \setminus [a, b]. \quad (1.1)$$

The function $w(z)Q_0(z)$ has the asymptotic series $\sum_{n=0}^{\infty} \mu_n z^{-n-1}$, $\mu_n = \int_a^b y^n w(y) dy$, as $z \rightarrow \infty$, [1,22]. When $[a, b]$ is bounded and $|z| > \max\{|a|, |b|\}$, the asymptotic series converges to $w(z)Q_0(z)$.

In this work we study properties of functions of the second kind of the Jacobi, big and little q -Jacobi polynomials as well as the symmetric Al-Salam–Chihara polynomials for $q > 1$. We also study the Askey–Wilson functions and the Wilson functions of the second kind. We show that both $\{P_n(x)\}$ and $\{Q_n(x)\}$ have the same raising and lowering operators. In particular, they have the same Rodrigues type formula, where $1 = P_0(x)$ is replaced by $Q_0(x)$. Both $P_n(x)$ and $Q_n(x)$ satisfy the same second order operator equation. This is known for Jacobi polynomials, and the little and big q -Jacobi polynomials. Our proofs are new in the known cases. We show how to find a closed form of Q_n from the knowledge of $w(x)Q_0(x)$ when the polynomials $\{P_n(x)\}$ satisfy a Rodrigues-type formula. We illustrate this new technique in the cases of the above mentioned polynomials. We also show how this ties up with the evaluation of integrals including the Nassrallah–Rahman integral [19] and some forms of q -beta integral, see [12, Section 2.10]. To the best of our knowledge this was never studied before. The moment problem and some spectral theory of these polynomials and their second order operators have been studied in [9,10], and [7].

Section 2 contains all the preliminary material needed in the rest of the paper. In Section 3, we treat the Jacobi polynomials where our technique can be easily explained without the technical details of the q -polynomials. Sections 4 and 5 are devoted to the big q -Jacobi and little q -Jacobi polynomials, respectively. Since the little q -Jacobi polynomials are limits of big q -Jacobi polynomials we include without proofs the corresponding results for the little q -Jacobi polynomials. Section 6 is devoted to the Askey–Wilson functions of second kind while Section 8 is devoted to the functions of the second kind associated with the Al-Salam–Chihara polynomials when $q > 1$. It is important to note that Section 7 contains the first example of functions of the second kind associated with an indeterminate moment problem. In Section 7 we discuss the general problem of the dependence of the function of the second kind on the orthogonality measure when the corresponding moment problem is indeterminate. We prove a general theorem about the structure of the function of the second kind corresponding to different solutions of the same moment problem, see Theorem 7.1.

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