

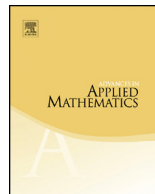


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A generalization of carries processes and Eulerian numbers

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ABSTRACT

We study a generalization of Holte's amazing matrix, the transition probability matrix of the Markov chains of the 'carries' in a *non-standard* numeration system. The stationary distributions are explicitly described by the numbers which can be regarded as a generalization of the Eulerian numbers and the MacMahon numbers. We also show that similar properties hold even for the numeration systems with the negative bases.

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1. Introduction and statements of results

The transition probability matrix so-called 'amazing matrix' of the Markov chain of the 'carries' has very nice properties [5], and has unexpected connection to the Markov chains of riffle shuffles [2, 3]. Diaconis and Fulman [3] studies a variant of the carries process, type *B* carries process. Novelli and Thibon studies the carries process in terms of noncommutative symmetric functions [7]. This paper studies a generalization of the carries process which includes Diaconis and Fulman's type *B* carries process as a special case. We study the transition probability matrices of the Markov chains of the carries in the numeration systems with non-standard digit sets. We show that the matrices have the eigenvectors which can be perfectly described by a generalization of Eulerian numbers and the MacMahon numbers [8,6,1,3]. We also show that similar properties hold even for the numeration systems with negative bases.

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$$\begin{array}{rcccccc}
 \cdots & C_4 & C_3 & C_2 & C_1 & C_0 \\
 \cdots & X_{1,4} & X_{1,3} & X_{1,2} & X_{1,1} & X_{1,0} \\
 \cdots & X_{2,4} & X_{2,3} & X_{2,2} & X_{2,1} & X_{2,0} \\
 \cdots & X_{3,4} & X_{3,3} & X_{3,2} & X_{3,1} & X_{3,0} \\
 \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 +) \cdots & X_{n,4} & X_{n,3} & X_{n,2} & X_{n,1} & X_{n,0} \\
 \hline
 \cdots & A_4 & A_3 & A_2 & A_1 & A_0
 \end{array}$$

Fig. 1. Carries process.

1.1. Numeration system

Throughout the paper, b denotes a positive integer and $\mathcal{D} = \{d, d + 1, \dots, d + b - 1\}$ denotes a set of integers containing 0. Therefore, $-b < d < b$. Then, we have a numeration system (b, \mathcal{D}) : Suppose that an integer x has a representation of the form,

$$x = (x_k x_{k-1} \cdots x_0)_b \stackrel{\text{def}}{=} x_0 + x_1 b + x_2 b^2 + \cdots + x_k b^k, \quad x_0, x_1, \dots, x_k \in \mathcal{D}, x_k \neq 0. \tag{1}$$

Then, it can be easily shown that this representation is uniquely determined for x and

$$\{(x_k x_{k-1} \cdots x_0)_b \mid k \geq 0, x_0, x_1, \dots, x_k \in \mathcal{D}\} = \begin{cases} \mathbb{Z} & d \neq 0, -b + 1, \\ \mathbb{N} & d = 0, \\ -\mathbb{N} & d = -b + 1 \end{cases}$$

is closed under the addition, where \mathbb{N} denotes the set of non-negative integers.

1.2. Carries process

Let $\{X_{i,j}\}_{1 \leq i \leq n, j \geq 0}$ be the set of independent random variables each of which is distributed uniformly over \mathcal{D} . Define the two stochastic processes (A_0, A_1, A_2, \dots) and (C_0, C_1, C_2, \dots) in the following way: $C_0 = 0$ with probability one. $(A_i)_{i \geq 0}$ is a sequence of \mathcal{D} -valued random variables satisfying

$$A_i \equiv C_i + X_{1,i} + \cdots + X_{n,i} \pmod{b}, \quad i = 0, 1, 2, \dots,$$

and

$$C_i = \frac{C_{i-1} + X_{1,i-1} + \cdots + X_{n,i-1} - A_{i-1}}{b}, \quad i = 1, 2, 3, \dots$$

(See Fig. 1.) It is obvious that (C_0, C_1, C_2, \dots) is a Markov process, which we call the carries process with n summands or simply n -carry process over (b, \mathcal{D}) .

1.3. A generalization of Eulerian numbers

Let $p \geq 1$ be a real number and n a positive integer. Then we define an array of numbers $v_{i,j}^{(p)}(n)$ for $i = 0, 1, \dots, n$ and $j = 0, 1, \dots, n + 1$ by

$$v_{i,j}^{(p)}(n) = \sum_{r=0}^j (-1)^r \binom{n+1}{r} [p(j-r) + 1]^{n-i}, \tag{2}$$

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