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On the application of difference potential theory to active noise control ☆

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Abstract

The application of the theory of difference potentials to the problem of active shielding and noise control is considered. Difference potential theory allows us to obtain the general solution of the problem in a finite-difference formulation. The solution is valid for arbitrary space domains, medium and boundary conditions. It only requires the information on the total sound (both "friendly" and "adverse") at the perimeter of the domain to be shielded. In contrast to the previous publications, in the current paper the mechanism of active shielding solution based on the difference potential theory is analysed. The theory of difference potentials is applied to the system of acoustic equations. The correspondence between the finite-difference solution and the continuous solution based on Green's function is shown for the case of a uniform medium. Different possible representations of the active shielding source terms are analysed. A clear physical interpretation of the optimal space step in the finite-difference solution is provided. The results can be important for both understanding the solution of the active shielding problem and practical applications.

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Keywords: Difference potentials; Acoustics; Active shielding; Noise control; Green's function

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1. Introduction

The problem of active noise control is a relatively new but extensively developed research activity in acoustics. In this problem, it is assumed that either some internal or external area is to be shielded via implementation of additional (secondary) sources, i.e. active shielding (AS). This is a key distinguishing feature of the approach from "passive" shielding where noise reduction is performed via mechanical means. Obviously, the mechanical route to shielding cannot be always implemented. In fact, often active and passive controls should be combined. The latter control may not be sufficient to filter low frequencies, while the former control is mostly appropriate for such frequencies. The problem becomes much more complicated however, if some "friendly" sound is assumed to be in the shielded area.

There are many publications devoted to the problem of AS and sound control. First theoretical papers in this field by Jessel, Malyuzhinets and Fedoryuk appeared about 30 years ago [4,7,15], while the first publications related to realistic practical applications appeared much later (see, e.g., [1,3]). Some of the noise suppression techniques are based on sound control in selected discrete [1–3,26] or directional [27] areas. Other techniques, in particular those developed by Kincaid et al. [9,10], assume detailed information regarding the sources and nature of noise. A number of publications are devoted to optimization of the strengths of spatially distributed secondary sources to minimize a quadratic pressure cost function [17,19]. The JMC method [8, 16,25], based on Huygens' principle, requires only the information on the undesirable field in the perimeter of the shielded domain. Yet this method cannot be used if a desirable field, generated in the shielded domain, has to be taken into account. The most comprehensive theoretical and practical reviews of the AS problem and its technical implementation can be found in books [5, 18,23] and report [25].

As mentioned above, generally in the standard approaches to AS, it is necessary to know the characteristics of "adverse" sources including their location. From a practical standpoint, this information is not often available. A separate class of methods requires the information on total sound (both "friendly" and "adverse") only at the perimeter of the domain to be shielded. It is very important to emphasize that the knowledge of both the "adverse" and "friendly" components is not necessarily required. Generally, these approaches are based on accessibility of Green's function [15,24] where the exact solution of the AS problem is obtained for the Helmholtz equation with constant coefficients. An original approach based on the Difference Potential Theory (DPT) [20,21] allows the ability to achieve the same end in a much more general formulation. This solution is applicable to arbitrary geometric configurations, medium and boundary conditions. There are only two principal requirements for its practical implementation. The problem must be linear and possess a unique solution. In contrast to the other approaches described above, the ultimate AS solution is achieved in a finite-difference form. From a practical standpoint this may not necessarily be treated as a drawback because the implementation of the AS assumes some discrete distribution of the AS sources. This approach has been analysed for application to the Helmholtz equation and its analogue with variable coefficients in [11–14]. A comprehensive analysis of continuous and finite-difference surface potentials mostly appropriate for the Helmholtz-type equation is carried out by Tsynkov in [24]. In [22] the solution of the linear AS problem is obtained in the continuous space for the acoustic equation with constant and variable coefficients. Whereas optimization of AS sources is studied in detail in the papers cited above, not much attention has been paid to the wave analysis of AS process itself.

It is to be noted that the general solution of the AS problem is generally applicable to 3D problems. Yet some important properties of the solution remain hidden unclear behind the math-