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Local energy-preserving algorithms for nonlinear fourth-order Schrödinger equation with trapped term

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ABSTRACT

Based on the rule that numerical algorithms should preserve the intrinsic properties of the original problem as many as possible, we propose two local energy-preserving algorithms for the nonlinear fourth-order Schrödinger equation with a trapped term. The local energy conservation law is preserved on any local time-space region. With appropriate boundary conditions, the first algorithm will be both globally charge- and energy-preserving and the second one will be energy-preserving. Numerical experiments show that the proposed algorithms provide more accurate solution than many existing methods and also exhibit excellent performance in preserving conservation laws.

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1. Introduction

High-order nonlinear Schrödinger equation with a trapped term (HNLSET) [1] is

$$i\frac{\partial u}{\partial t}+(-1)^m\alpha\frac{\partial^{2m}u}{\partial x^{2m}}+\frac{\partial\hbar(|u|^2)}{\partial|u|^2}u+g(x)u=0, \tag{1}$$

where $i=\sqrt{-1}$, $m\in\mathbb{N}$, α is a constant, $\hbar(|u|^2)$ is a bounded real differentiable functional of $|u(x,t)|^2$ and $g(x)$ is a real-valued bounded function. The equation describes many physical phenomena and has been widely applied to many important physical contexts, such as fluid dynamics, nonlinear optics, quantum physics, plasma physics and Bose-Einstein condensates. For $m=1$ and $g(x)=0$, it degenerates to a nonlinear Schrödinger equation, which has been studied by many authors analytically and numerically [2–7].

In this paper, we consider the fourth-order nonlinear Schrödinger equation with a trapped term (FNLSET)

$$iu_t+u_{xxxx}+\hbar'(|u|^2)u+g(x)u=0,\quad \hbar(|u|^2)=\rho|u|^4, \tag{2}$$

imposed on initial and periodic boundary conditions

$$u(x,0)=u_0(x),\quad \partial_x^s u(x,t)=\partial_x^s u(x+L,t),\ (x,t)\in[0,L]\times[0,T], \tag{3}$$

where $s=0,1,2,3$. The potential term $g(x)$ is to localize the wave around the origin and the equation with $g(x)=0$ is used to study the effect of small fourth-order dispersion in the propagation of intense laser beam in a bulk medium [8–10]. Eq. (2) admits a charge conservation law

$$\mathcal{Q}(t)=\int_0^L|u(x,t)|^2dx=\int_0^L|u(x,0)|^2dx=\mathcal{Q}(0), \tag{4}$$

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and an energy conservation law

$$\mathcal{E}(t)=\int_0^L\big(|u_{xx}|^2+\hbar(|u|^2)+g(x)|u|^2\big)dx=\mathcal{E}(0). \tag{5}$$

There have been some numerical works for the FNLSET equation. In [11], Chao proposed an energy-conserving difference scheme

$$i\frac{u_j^{n+1}-u_j^n}{\tau}+\bar{\delta}_x^4u_j^{n+\frac12}+\frac{\hbar(|u_j^{n+1}|^2)-\hbar(|u_j^n|^2)}{|u_j^{n+1}|^2-|u_j^n|^2}u_j^{n+\frac12}+g(x_j)u_j^{n+\frac12}=0, \tag{6}$$

where $\bar{\delta}_x^4$ is the second-order central difference operator corresponding to ∂_x^4, τ represents time step and h represents space step. The scheme also conserves the discrete version of (4) exactly. In [12], Zeng proposed a leap-frog difference scheme which is explicit and conditionally stable with a severe CFL condition $\tau/h^4\le 1/16$. He also proposed a series of symplectic schemes in [13]. Kong et al. proposed symplectic schemes [14]

$$i\frac{u_j^{n+1}-u_j^n}{\tau}+\bar{\delta}_x^4u_j^{n+\frac12}+\hbar'(|u_j^{n+\frac12}|^2)u_j^{n+\frac12}+g(x_j)u_j^{n+\frac12}=0, \tag{7}$$

and

$$\begin{aligned}&i\frac{u_j^{n+1}-u_j^n}{\tau}+\hbar'(|u_j^{n+\frac12}|^2)u_j^{n+\frac12}+g(x_j)u_j^{n+\frac12}\\&-\frac{1}{6h^4}(u_{j-3}^{n+\frac12}-12u_{j-2}^{n+\frac12}+39u_{j-1}^{n+\frac12}-56u_j^{n+\frac12}+39u_{j+1}^{n+\frac12}-12u_{j+2}^{n+\frac12}+u_{j+3}^{n+\frac12})=0,\end{aligned} \tag{8}$$

for Eq. (2). We denote the two schemes as **KS1** and **KS2**, respectively. The two schemes preserve the discrete charge conservation law, but do not preserve the energy conservation law. In [15], Hong and Kong studied the FNLSET equation with $\hbar(|u|^2)=3|u|^4$ and $g(x)=-150\sin^2x$. They derived a multi-symplectic Runge–Kutta (**MSRK**) scheme and a multi-symplectic Fourier pseudospectral (**MSFS**) scheme

$$i\frac{u^{n+1}-u^n}{\tau}+D_1^4u^{n+\frac12}+g(x)u^{n+\frac12}+\hbar'(|u^{n+\frac12}|)u^{n+\frac12}=0, \tag{9}$$

where D_1 is a first-order spectral differential matrix. In the same paper, by using the idea of split-step technique, they also proposed a SSMS scheme

$$\begin{aligned}&i\frac{u_j^*-u_j^n}{\tau/2}=\frac12(150\sin^2x_j-1.5|u_j^n+u_j^*|^2)(u_j^n+u_j^*),\\&\frac{i}{16}\delta_t(u_{j-2}^*+4u_{j-1}^*+6u_j^*+4u_{j+1}^*+u_{j+2}^*)+\bar{\delta}_x^4\widehat{u}_j=0,\\&i\frac{u_j^{n+1}-u_j^{**}}{\tau/2}=\frac12(150\sin^2x_j-1.5|u_j^{n+1}+u_j^{**}|^2)(u_j^{n+1}+u_j^{**}),\end{aligned} \tag{10}$$

where $\widehat{u}_j=(u_j^*+u_j^{**})/2$ and $\delta_tu_j^*=(u_j^{**}-u_j^*)/\tau$. The three methods in [15] are charge-preserving, but not energy-conserving.

As we know, it is preferable that numerical schemes have discrete analogues of the physical conservation properties, since they often yield physically correct results and also numerical stability. However, many energy-preserving algorithms for partial differential equations (PDEs) may bring the constraints and inconvenience since they inevitably depends on the boundary conditions. That is to say, the necessary conditions for applying an energy-preserving algorithms to a given PDEs are not only the conservative system itself, but also the appropriate boundary conditions. The energy-preserving algorithms will be invalid for the problem with inappropriate boundary conditions. Therefore, how to construct algorithms preserving the energy conservation law in any local time-space domain are interesting. Wang et al. proposed the local structure-preserving algorithms (LSPAs) for the Klein–Gordon equation in [16], and later we proposed LSPAs for the coupled nonlinear Schrödinger system [17] and multi-symplectic PDE [18].

Let $u(x,t)=p(x,t)+iq(x,t)$, where $p(x,t)$ and $q(x,t)$ are real-valued functions. By introducing some variables, the FNLSET equation can be recast as a series of ordinary differential equations (ODEs)

$$\begin{cases}-q_t=-m-g(x)p-\hbar'(|u|^2)p,\\ p_t=-d-g(x)q-\hbar'(|u|^2)q,\end{cases} \tag{11}$$

$$\begin{cases}\varphi_x=\psi,\\ p_x=\varphi,\end{cases}\quad\begin{cases}\eta_x=m,\\ \psi_x=\eta,\end{cases}\quad\begin{cases}\alpha_x=\beta,\\ q_x=\alpha,\end{cases}\quad\begin{cases}r_x=d,\\ \beta_x=r.\end{cases} \tag{12}$$

Proposition 1. *The systems (11) and (12) possess a local energy conservation law (LECL)*

$$\partial_x(2(\eta p_t+rq_t-\psi\varphi_t-\beta\alpha_t))+\partial_t\big(|u_{xx}|^2+g(x)|u|^2+\hbar(|u|^2)\big)=0. \tag{13}$$

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