Applied Mathematics and Computation 295 (2017) 53–64

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A finite frequency approach to control of Markov jump linear systems with incomplete transition probabilities

Mouquan Shen [a,b,*], Dan Ye [c,*]

[a] College of Electrical Engineering and Control Science, Nanjing Technology University, Nanjing 211816, China
[b] Key Laboratory of Advanced Control and Optimization for Chemical Processes, Ministry of Education, East China University of Science and Technology, Shanghai 200237, China
[c] College of Information Science and Engineering, Northeastern University, Shenyang 110819, China

ARTICLE INFO

Keywords:
Markov jump systems
Finite frequency domain
Linear matrix inequality

ABSTRACT

This paper is concerned with the state feedback control of continuous Markov jump linear systems with incomplete transition probabilities in finite frequency domain. By developing a new technique to handle the coupling among Lyapunov variable, system matrix and controller parameter, new sufficient conditions for the closed-loop system to be stochastically stable with the required finite frequency performance are established in terms of linear matrix inequalities. Meanwhile, the finite frequency state feedback controller is also obtained by the proposed conditions directly. The validity of the proposed method is demonstrated by a numerical example.

1. Introduction

The Markov jump system is categorized as a class of stochastic hybrid system. Due to its widespread application in the fields of economics, robotics, and wireless communications, much effort has been devoted to this field [1]. On the hypothesis of known transition probabilities, fruitful results on stability, stabilization, sliding mode control, H_2 and H_∞ control and filtering are referred to [2–15]. Although the hypothesis is apt to do analysis and synthesis in theory, it is difficult to employ the obtained results to practical engineering problems. To shorten the gap between theory and practical applications, uncertain transition probabilities are assumed to be norm bounded or polytope and handled by the robust methodology [16,17]. In this situation, the structure and nominal terms of these uncertainties should be known in advance [18]. To approximate the realistic situation, transition probabilities are allowed to partly known [18]. With the partly known transition probabilities presented in [18], a multiple integral approach is employed to cast the impulsive synchronization of Markovian jumping neural networks in [19] and stochastic sampled-date based exponential synchronization of Markovian jumping neural networks is discussed in [20]. To reduce the possible conservativeness incurred by unknown transition probabilities, a free weighting matrix method is built in [21] and the property of transition probabilities are made thoroughly in [22].

On the other hand, the frequency characteristic of disturbance may be known beforehand, especially in mechanical systems. Consequently, in the course of controller design for mechanical systems, integrating the frequency characteristic could enhance the closed-loop system performance [23,24]. Taking into account this point, H_∞ control, fault detection and model reduction in finite frequency domain for linear systems are given in [25–29]. What's more, finite frequency fuzzy filtering

* Corresponding author at: College of Electrical Engineering and Control Science, Nanjing Technology University, Nanjing, 211816, China.
E-mail addresses: mouquanshen@gmail.com (M. Shen), yedan@ise.neu.edu.cn (D. Ye).

http://dx.doi.org/10.1016/j.amc.2016.09.024
0096-3003/

of nonlinear systems is addressed in [30] and H_∞ state feedback control of fuzzy system in finite frequency is presented in [31]. Regarding to Markov jump system, only few results are available in the literature [32,33]. Specifically, modeling the accessability of each node to the shared channel as a Markov chain, a geometric scheme based fault detection and isolation for the resultant discrete Markov jump system in finite frequency domain is proposed in [32]. Based on the generalized Kalman–Yakubovich–Popov lemma, stochastic consensus control of continuous Markov jump system with middle frequency specification is discussed in [33]. Nevertheless, transition probabilities in these results are required to be known. Once they are unknown, the proposed approaches are out of use due to nonlinearities induced by them. Moreover, the technique to deal with finite frequency still has room to be further improved.

Stimulated by the above mentioned points, this paper is devoted to state feedback control of Markov jump linear systems with finite frequency disturbances. Incomplete transition probabilities contain known, uncertain and unknown. Instead of using the generalized Kalman–Yakubovich–Popov lemma directly, enlightening by the time domain inequalities proposed in [24], a finite frequency performance for the considered Markov jump linear system is defined firstly. Then, nonlinearities induced by unknown transition probabilities are conquered by the transition matrix property. Based on the definition and a new decoupling measurement, sufficient conditions for the closed-loop system to be stochastically stable with the required finite frequency performance are established in the framework of linear matrix inequalities. With the help of Finsler lemma, the parameter dependent Lyapunov function approach is adopted to ensure the closed-loop system to be stochastically stable and meet the required finite frequency performance. Compared with the existing method to get the controller gain, the proposed method has no need to pre- and post-multiply an inverse matrix. A single-link robot arm is simulated to show the fact that the finite frequency performance is less conservativeness than that of the full frequency.

This article is organized as follows. System model, definition and technical lemma are introduced in Section 2. An effective finite frequency controller design method is proposed in Section 3. A single-link robot arm is given in Section 4 to verify the validity of the proposed approach. Lastly, Section 5 concludes the paper.

Notation: Throughout the paper, the notation $R > 0$ (< 0) means that R is symmetric and positive (negative) definite. $\mathbb{R}^n$ indicates the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $\mathbb{L}_2$ means the space of square integrable vector functions over $[0\ \ \infty)$ with norm $||x||_2 = \{\int_0^\infty x^T(t)x(t)dt\}^{\frac{1}{2}}$. The transpose of M is denoted by M^T. $\star$ stands for the entries of matrices implied by symmetry. Then, symbols $sym(X)$ and $He(X)$ are employed to represent $X + X^T$ and $\frac{(X+X^T)}{2}$ respectively. What is more, matrices, if not explicitly stated, are assumed to have compatible dimensions. $\mathbb{E}$ denotes the expectation operator. $\mathcal{H}^\perp$ is the kernel of $\mathcal{H}$. j is the imaginary unit.

2. Problem statement and preliminaries

Consider the following continuous Markov jump linear system as

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + B_1(r(t))u(t) + B_2(r(t))w(t) \\ z(t) = C(r(t))x(t) + D_1(r(t))u(t) + D_2(r(t))w(t) \end{cases} \tag{1}$$

where $x(t)$, $u(t)$ and $w(t)$ are continuous system state vector, control input and the energy bounded noise with known frequency respectively. $z(t)$ is the measured output. $A(r(t))$, $B_1(r(t))$, $B_2(r(t))$, $C(r(t))$, $D_1(r(t))$, $D_2(r(t))$ are system matrices with approximate dimension. $r(t)(t \geq 0)$ is continuous Markov process belonging to a finite set $\mathcal{I} = \{1, 2, \ldots, N\}$. The mode transition probabilities of continuous case $r(t)$ satisfies

$$Pr(r(t+h) = l|r(t) = i) = \begin{cases} \pi_{il}h + o(h), & if \quad l \neq i \\ 1 + \pi_{ii}h + o(h), & if \quad l = i \end{cases}$$

where $h > 0$ and $\lim_{h\to 0} \frac{o(h)}{h} = 0$, $\pi_{il} \geq 0$ $(i, l \in \mathcal{I}, l \neq i)$ represents the transition probability from mode i to mode l and $\pi_{ii} = -\sum_{l\neq i}^N \pi_{il}$.

As discussed in the existing result [22], transition probabilities in this paper are also deemed to be incomplete. For instance, the incomplete transition probability matrix for system (1) with four modes is

$$\begin{bmatrix} \pi_{11} & ? & \alpha_{13} & ? \\ ? & ? & ? & ? \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \\ \pi_{41} & ? & ? & \pi_{44} \end{bmatrix} \tag{2}$$

where ? means that the corresponding elements are inaccessible and α_{ij} is uncertain with known lower and upper bounds $(\underline{\alpha}_{ij}, \bar{\alpha}_{ij})$. To formulate the accessability of transition probability concisely, two sets are utilized to cover all above information

$$\begin{cases} \mathcal{I}_k^i = \{l|\pi_{il} \quad is \quad known \quad or \quad uncertain, \quad l \in \mathcal{I}\} \\ \mathcal{I}_{uk}^i = \{l|\pi_{il} \quad is \quad unknown, l \in \mathcal{I}\} \end{cases} \tag{3}$$

Remark 1. In [18], uncertain transition probabilities are treated as unknown. Obviously, this treatment could be conservative when all transition probabilities are uncertain with known bounds. To fill up this deficiency, the above set $\mathcal{I}_k^i$ includes uncertain ones.