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Distributed finite-time leaderless consensus control for double-integrator multi-agent systems with external disturbances

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ABSTRACT

We study the finite-time consensus control of double-integrator multi-agent systems with external bounded disturbances under the fixed leaderless network communication topology. Based on the relative position and the relative velocity information, a novel finite-time consensus protocol is constructed. By using the Lyapnuov finite-time theory, it is shown that the proposed protocol can coordinate the states of agents to guarantee the finite-time consensus under an undirected connected topology. And then we extend the results onto the strongly connected directed communication topology. Importantly, the finite convergence time of the leaderless systems is explicitly presented. Finally, the effectiveness of the results is illustrated by numerical simulation.

1. Introduction

In recent years, the study of consensus in multi-agent systems has been hot topic with application to the coordination of multi-agent systems, including formation flight of unmanned air vehicles(UAVs), clusters of satellites, self-organization, automated highway systems, and congestion control in communication networks [1]. It is well known that the consensus of multi-agent systems have several advantages, including increased feasibility, accuracy, robustness, flexibility, cost, energy efficiency, and probability of success, etc. The objective of consensus problem is to design a proper distributed control algorithms such that a group of agents reach an agreement on some state of interest. Consensus has been investigated for multi-agent systems with various distributed control strategies, including adaptive control [2], sliding mode control [3,4], Lyapunov control [5], optimal control [6], robust H_∞ control [7], pining control [8,9] etc.

Various control strategies have been studied for different multi-agent systems with various scenarios. According to the number of leaders existing in the systems, consensus can be classified into leaderless consensus, distributed tracking with one leader and containment control with multiple leaders. Recently, a variety of distributed consensus algorithms have been proposed to solve the leaderless consensus problems under different scenarios [10–13]. The leaderless consensus control problems have been investigated for first-order multi-agent systems in the presence of limited and unreliable information exchange with dynamically changing interaction topologies [10]. The leaderless consensus was studied for first-order nonlinear multi-agent systems under a directed communication network [11]. Typically, second-order consensus problem aims at studying how to make the agents move with the same velocity and converge to the same position. A new distributed

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observer-type consensus protocol has been proposed to study the second-order linear multi-agent systems with general linear node dynamics under the directed topology [12]. Wen et al.8 have investigated the consensus for a second-order integrator systems with a fixed directed topology and communication constraints, where each agent is assumed to share information only with its neighbors on some disconnected time intervals [13]. Some necessary and sufficient condition are presented to ensure the consensus of the second-order nonlinear systems [14,15], where the steady state of velocities is time-invariant in undirected and directed topology networks.

It is noted that the above references mainly focus on the asymptotic consensus control for multi-agent systems. In fact, the convergence rate is a significant performance index for evaluating the effectiveness of a distributed consensus control of multi-agent systems. Compared with asymptotical consensus convergence, the finite-time convergence has faster convergence rates and higher robustness. Finite-time consensus for multi-agent systems with first-order and second-order dynamics was studied in [16–19], respectively. Numerous researchers can improve the convergence rate by enlarging the coupling strength, optimizing the control gain or choosing better communication topology [20–23]. With the aid of sliding-mode control technique, distributed finite-time tracking controls for first-order nonlinear systems and second-order integrator systems have been investigated [24,25], respectively. By using the sliding-mode control technique, containment control for second-order heterogeneous system and nonlinear systems with external disturbances has been studied [26,27]. By using the homogenous systems theory, the finite-time consensus tracking for a second-order nonlinear systems have been considered [28]. But, the precise time was not yet obtained. Meng et al. proposed distributed finite-time observers protocol for double integrator systems with a time-varying leader's velocity with bounded external disturbance [29].

To our knowledge, most results of the finite-time consensus control deal with the leader-following networks. However, there is no good result on the finite-time consensus problems of leaderless systems. Recently, a saturated protocol is proposed based on both relative position and relative velocity measurements for solving the finite-time consensus problem of second-order leaderless systems [30]. This is based on the homogenous theory, so the setting time is not present. Here, we will expand the existing some results [13–15], the consensus are derived for second-order integrator systems and nonlinear systems by using the algebraic graph theory, matrix theory, and Lyapunov control approach. In particular, the distributed finite-time consensus control of the double-integrator leaderless systems with external bounded disturbances will be investigated in the undirected and directed communication topology networks. A novel consensus protocol is designed to coordinate the states of agents to converge to consensus in finite time under a connected undirected topology. And then, we will extend the results for the strongly connected directed topology. Compared with the existing results, one of the important innovation of this paper is that we extend the finite-time consensus problems from leader-following networks to leaderless systems, and the setting time of the leaderless system is explicitly presented.

The rest of this paper is organized as follows. Section 2 introduces some useful preliminaries of graph theory and the model description. In Section 3, the finite-time consensus control for second-order systems under the connected undirected graph is discussed, and we extend the results to the strongly connected communication graph. In Section 4, a numerical example is given to verify the theoretical analysis. Section 5 presents main conclusion.

2. Preliminaries and model description

2.1. Notations

Let R and C be the sets of real and complex numbers, respectively. **1** and **0** represents the vector with all entries being one and the vector with all entries being zero, respectively. I_n is the n -dimensional identity matrix. $A > 0$ ($A < 0$) means that the matrix A is positive (negative) definite. $\|A\|_\infty$ denotes the $\infty-$norm of matrix A. $\lambda_{min}(A)$ and $\lambda_{max}(A)$ represents the minimal eigenvalue and the maximum eigenvalue of the matrix A, respectively. Given a vector $\xi = [\xi_1, \xi_2, \ldots, \xi_p]^T$ and constant $\kappa > 0$, define $\mathrm{sig}(\xi)^\kappa = [\mathrm{sign}(\xi_1)|\xi_1|^\kappa, \mathrm{sign}(\xi_2)|\xi_2|^\kappa, \ldots, \mathrm{sign}(\xi_p)|\xi_p|^\kappa]^T$ and $\mathrm{sign}(\xi) = [\mathrm{sign}(\xi_1), \mathrm{sign}(\xi_2), \ldots, \mathrm{sign}(\xi_p)]^T$, where sign(.) is the signum function. Let $\mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_p)$ represent a diagonal matrix with diagonal elements ξ_1, ξ_2, $\cdots$, ξ_p. The Kronecker product of matrices $A \in R^{m\times n}$ and $B \in R^{p\times q}$ is defined as

$$A \otimes B = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{pmatrix}.$$

Lemma 1 (*[31]*)**.** *Given matrices A, B, C, and D with compatible dimensions, we have the following results,*

(1) $(A\otimes B)(C\otimes D) = (AC)\otimes(BD)$,
(2) $(A\otimes B) + (A\otimes C) = A\otimes(B+C)$,
(3) $(A\otimes B)^{-1} = A^{-1}\otimes B^{-1}$, $(A\otimes B)^T = A^T\otimes B^T$,
(4) *if A and B are symmetric positive definite, so is A⊗B.*

Lemma 2 (*[32]*)**.** *The following linear matrix inequality(LMI)*

$$\begin{pmatrix} M_1(x) & M_2(x) \\ M_2(x)^T & M_3(x) \end{pmatrix} > 0,$$