# A block active set algorithm with spectral choice line search for the symmetric eigenvalue complementarity problem 

Carmo P. Brás ${ }^{\text {a,* }}$, Andreas Fischer ${ }^{\text {b }}$, Joaquim J. Júdice ${ }^{\text {c }}$, Klaus Schönefeld ${ }^{\text {b }}$, Sarah Seifert ${ }^{\text {b }}$<br>${ }^{\text {a }}$ Centro de Matemática e Aplicações (CMA), FCT, Universidade NOVA de Lisboa, 2829-516, Caparica, Portugal<br>${ }^{\mathrm{b}}$ Institute of Numerical Mathematics, Technische Universität Dresden, 01062, Dresden Germany<br>${ }^{\text {c }}$ Instituto de Telecomunicações, Universidade de Coimbra - Polo II, 3030-290, Coimbra, Portugal

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#### Abstract

In this paper, we address the solution of the symmetric eigenvalue complementarity problem (EiCP) by treating an equivalent reformulation of finding a stationary point of a fractional quadratic program on the unit simplex. The spectral projected-gradient (SPG) method has been recommended to this optimization problem when the dimension of the symmetric EiCP is large and the accuracy of the solution is not a very important issue. We suggest a new algorithm which combines elements from the SPG method and the block active set method, where the latter was originally designed for box constrained quadratic programs. In the new algorithm the projection onto the unit simplex in the SPG method is replaced by the much cheaper projection onto a box. This can be of particular advantage for large and sparse symmetric EiCPs. Global convergence to a solution of the symmetric EiCP is established. Computational experience with medium and large symmetric EiCPs is reported to illustrate the efficacy and efficiency of the new algorithm.


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## 1. Introduction

The eigenvalue complementarity problem (EiCP) consists in finding $(\lambda, x) \in \mathbb{R} \times \mathbb{R}^{n} \backslash\{0\}$ satisfying

$$
\begin{equation*}
(\lambda B-A) x \geq 0, \quad x \geq 0, \quad x^{\top}(\lambda B-A) x=0, \tag{1}
\end{equation*}
$$

where $A, B \in \mathbb{R}^{n \times n}$ are given matrices with $B$ being positive definite. Equivalently, we may seek $(\lambda, x) \in \mathbb{R} \times \mathbb{R}^{n}$ so that

$$
\begin{equation*}
(\lambda B-A) x \geq 0, \quad x \geq 0, \quad x^{\top}(\lambda B-A) x=0, \quad e^{\top} x=1 \tag{2}
\end{equation*}
$$

holds with $e:=(1, \ldots, 1)^{\top} \in \mathbb{R}^{n}$. If a pair $(\lambda, x)$ solves EiCP, then $\lambda$ is called complementary eigenvalue and $x$ complementary eigenvector associated to $\lambda$. The EiCP has been introduced in [31] as a generalization of the classical eigenvalue problem to a closed and convex cone. To our knowledge, a first interesting engineering application of the EiCP was described in [28]. In recent years, many other papers appeared in the literature describing theoretical results, algorithms, and applications for EiCPs as well as some extensions [1,2,9,10,17,18,20-22,25,27,29,30,32,33,38].

[^0]In this paper, we are dealing with symmetric EiCPs [30], i.e., the matrices $A$ and $B$ are both symmetric. For any $x \in$ $\mathbb{R}^{n} \backslash\{0\}$, let

$$
f(x):=-\frac{x^{\top} A x}{x^{\top} B x}
$$

denote the negative generalized Rayleigh quotient associated to EiCP (1). It was shown in $[30,34]$ that any stationary point $x^{*}$ of the minimization problem

$$
\begin{equation*}
f(x) \rightarrow \min \quad \text { s.t. } \quad x \in \Delta:=\left\{x \in \mathbb{R}^{n} \mid e^{\top} x=1, x \geq 0\right\} \tag{3}
\end{equation*}
$$

with $\lambda^{*}:=-f\left(x^{*}\right)$ provides a solution of (2), and vice versa. This problem can also be regarded as a fractional quadratic program over the unit simplex $\Delta$. Nonlinear programming algorithms [26] may be used to solve the symmetric EiCP by computing a stationary point of (3). In particular, the use of the spectral projected-gradient (SPG) method [5,6] was suggested in [20]. In each step of the SPG method the direction $-\eta \nabla f(x)$ equipped with the spectral choice line search parameter $\eta$ [4] is projected onto the simplex $\Delta$. In this way, a feasible descent direction is obtained and $f$ is minimized along this direction. This projection onto $\Delta$ has the worst-case complexity of $O\left(n^{2}\right)$ per step and can be done quite efficiently by one of the methods described in [19,37]. The SPG method incorporating one of these projection techniques was reported [20] to perform well for large-scale EiCPs, particularly when the accuracy of the computed solution is not a very important issue.

In this paper, we introduce a new algorithm which combines ideas from the SPG method and from the block active set (BAS) method described in [16]. The new spectral block active set algorithm (Spectral BAS) employs in each iteration a block active set strategy for forecasting the active set at some stationary point. Several components of the search direction are determined in this way. The remaining components of the search direction are computed by the SPG strategy mentioned before. However, instead of projecting onto $\Delta$, only a projection onto the nonnegative orthant is needed. An exact line search and a normalization step complete the algorithm and guarantee that the iterates stay within the simplex $\Delta$. Hence, the Spectral BAS algorithm only requires cheap projections of complexity $O(n)$ onto the nonnegative orthant, which can be of particular interest for large-scale sparse EiCPs. Computational experience reported in this paper shows that the algorithm seems to be quite efficient for the solution of symmetric EiCPs associated to the maximum clique problem [7]. This may have important implications on the design of new algorithms for this difficult problem which are based on the solution of EiCPs. The algorithm is also efficient to solve symmetric EiCPs with unstructured sparse matrices if the accuracy is not too at stake, but may face some difficulties for getting more accurate solutions. A new preprocessing technique is also introduced that improves the efficiency of the Spectral BAS algorithm in practice. Furthermore, the Spectral BAS algorithm is competitive with the SPG method and seems to be more efficient for large-scale EiCPs. This is mainly due to the cheaper projection technique used by the Spectral BAS algorithm.

The paper is organized as follows. We briefly review a characterization of stationary points of the minimization problem (3) in Section 2. Then, Section 3 provides a detailed description of the new Spectral BAS algorithm including its welldefinedness and important basic properties. Global convergence of this algorithm is shown in Section 4. Computational experience with the Spectral BAS algorithm and a comparison with the SPG method are reported in Section 5 . Finally, some conclusions are included in the last section of the paper.

Notation. For a vector $x \in \mathbb{R}^{n}$ and an index set $J \subset I:=\{1, \ldots, n\}$ the vector $x_{J}$ consists of all components $x_{j}$ of $x$ with $j \in J$. The Euclidean projection of some $z \in \mathbb{R}^{n}$ onto the simplex $\Delta$ is denoted by $P_{\Delta}(z)$. If $y \in \mathbb{R}^{q}$ is projected onto the nonnegative orthant $\mathbb{R}_{+}^{q}$ we simply write $y_{+}$to denote the result of this projection.

## 2. Properties of the fractional program

The unit simplex $\Delta$ is a nonempty, closed, and convex set. Therefore, $x^{*} \in \Delta$ is a stationary point of (3) if and only if the first-order necessary optimality condition for (3) are satisfied at $x^{*}$, i.e., if

$$
\begin{equation*}
\nabla f\left(x^{*}\right)^{\top}\left(x-x^{*}\right) \geq 0 \quad \text { for all } x \in \Delta \tag{4}
\end{equation*}
$$

By the positive definiteness of $B$, the function $f: \mathbb{R}^{n} \backslash\{0\} \rightarrow \mathbb{R}$ is continuously differentiable with

$$
\begin{equation*}
\nabla f(x)=\frac{2}{x^{\top} B x}\left(\frac{x^{\top} A x}{x^{\top} B x} B x-A x\right) \tag{5}
\end{equation*}
$$

and it can be easily verified that

$$
\begin{equation*}
x^{\top} \nabla f(x)=0 \quad \text { for all } x \in \mathbb{R}^{n} \backslash\{0\} \tag{6}
\end{equation*}
$$

Thus, instead of (4), a stationary point $x^{*}$ of (3) can be characterized by $x^{*} \in \Delta$ and

$$
\begin{equation*}
\nabla f\left(x^{*}\right)^{\top} x \geq 0 \quad \text { for all } x \in \Delta \tag{7}
\end{equation*}
$$

Because all vectors of the canonical basis of $\mathbb{R}^{n}$ belong to $\Delta$, condition (7) implies $\nabla f\left(x^{*}\right) \geq 0$. This, $x^{*} \geq 0$, and (6) yield that $x^{*}$ is stationary for (3) if and only if $x^{*}$ belongs to $\Delta$ and satisfies

$$
\begin{equation*}
\min \{x, \nabla f(x)\}=0 \tag{8}
\end{equation*}
$$

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[^0]:    * Corresponding author.

    E-mail addresses: mb@fct.unl.pt (C.P. Brás), Andreas.Fischer@tu-dresden.de (A. Fischer), joaquim.judice@co.it.pt (J.J. Júdice), Klaus.Schoenefeld@tudresden.de (K. Schönefeld).

