



Delay-dependent stability analysis of neural networks with time-varying delay: A generalized free-weighting-matrix approach[☆]



Chuan-Ke Zhang^{a,b,*}, Yong He^a, Lin Jiang^b, Wen-Juan Lin^a, Min Wu^a

^aSchool of Automation, China University of Geosciences, Wuhan 430074, China

^bDepartment of Electrical Engineering & Electronics, University of Liverpool, Liverpool L69 3GJ, United Kingdom

ARTICLE INFO

Keywords:

Neural networks

Time-varying delay

Generalized free-weighting-matrix approach

Stability

ABSTRACT

This paper investigates the delay-dependent stability problem of continuous neural networks with a bounded time-varying delay via Lyapunov–Krasovskii functional (LKF) method. This paper focuses on reducing the conservatism of stability criteria by estimating the derivative of the LKF more accurately. Firstly, based on several zero-value equalities, a generalized free-weighting-matrix (GFWM) approach is developed for estimating the single integral term. It is also theoretically proved that the GFWM approach is less conservative than the existing methods commonly used for the same task. Then, the GFWM approach is applied to investigate the stability of delayed neural networks, and several stability criteria are derived. Finally, three numerical examples are given to verify the advantages of the proposed criteria.

© 2016 Elsevier Inc. All rights reserved.

1. Introduction

Neural networks have been successfully applied in image processing, pattern recognition, associative memory, optimization problem, etc. [1,2,3]. For those applications, the artificial neural networks usually must be stable [6]. However, the finite switching speed of amplifiers and the inherent communication time between neurons inevitably cause time delays during the implementation of artificial neural networks [7]. These delays might lead to undesired dynamics like oscillation and instability. Therefore, it is an important job to determine the admissible maximal delay bound (AMDB) such that the delayed neural networks (DNNs) with a delay less than this bound remain stable. In the past few decades, such delay-dependent stability analysis problem has become a hot issue in the field of neural networks [4].

These delays are usually time-varying and Lyapunov–Krasovskii functional (LKF) method can easily handle such DNNs, thus, the LKF method has become the one of most popular methods for stability analysis and finding the AMDBs. The DNNs with bounded delays are asymptotically stable if there exists an LKF, which consists of state vector based quadratic terms, is positive definite and has a negative gradient over the time. Linear matrix inequalities (LMIs) based delay-dependent stability criteria can be easily used to check whether or not such LKF can be found for the DNNs. However, the criteria derived have some conservatism since they are only sufficient conditions. In order to find the AMDBs more accurately, one important issue in the related research is to develop new criteria with less conservatism. This paper also investigates the stability of DNNs following this direction.

[☆] This work is supported partially by the National Natural Science Foundation of China under grant nos. 61503351, 51428702, and 61304011, and the Hubei Provincial Natural Science Foundation of China under grant 2015CFA010.

* Corresponding author. Fax: +86(0)2787175060.

E-mail address: ckzhang@cug.edu.cn (C.-K. Zhang).

<http://dx.doi.org/10.1016/j.amc.2016.08.043>

0096-3003/© 2016 Elsevier Inc. All rights reserved.

By constructing a special form of LKF candidate, tractable LMI-based stability criteria are derived using necessary techniques to estimate the LKF and its derivative. Therefore, the construction and the treatment for the LKF are the basic issues related to how conservative the criteria are.

In the early research, the LKFs for the stability analysis of the DNNs were constructed by introducing delay-based single and double integral terms into the typical non-integral quadratic form of Lyapunov function for delay-free systems [5,6,8–14]. Later, researchers have developed many new LKFs by making the previous ones more general in three aspects.

- (1) Firstly, based on several subintervals divided from the whole delay region, some scholars have developed the delay-partition-based LKFs by replacing the original integral terms with multiple new integral terms with smaller domain of integration [16–30].
- (2) Secondly, by using various state vectors (current, delayed, and/or integrated state vectors *etc.*), some scholars have developed new LKFs by augmenting the quadratic terms of original LKFs [31–42].
- (3) Thirdly, since the triple integral term was found to be helpful for reducing the conservatism of stability criteria for linear time-delay systems [47], similar and/or extended forms have also been widely applied to stability analysis of various DNNs [49–58].

Although those LKFs have different forms, they all include a common term with the form of $\int_{-h}^0 \int_{t+\theta}^t y^T(s) R y(s) ds d\theta$ (Here $h > 0$ is the scalar, $y(s)$ is the system state-based vector, and matrix $R > 0$). Then its derivative contains the term as follows

$$- \int_{t-h}^t y^T(s) R y(s) ds \quad (1)$$

This term was directly dropped in the early literature [5], but such treatment is very conservative. Later, this term was retained to improve the results, in which case it must be estimated to represent the criterion in the form of tractable LMI. As mentioned in [43], the estimation of the above single integral term is strongly related to the conservatism of criteria. Therefore, the stability criteria of DNNs have been improved gradually by using more effective techniques for this estimation task.

The basic inequality was used to estimate the single integral term [6]. Since He et al. [44] proposed the free-weighting matrix (FWM) approach, which is more effective than the basic inequality, the FWM approach has been widely used in the stability analysis of DNNs [8–13,28–32]. The slack matrices introduced by the FWM approach provide great freedom of the criteria.

An alternative method that estimates the original integral terms directly was also used in the stability analysis of DNNs. The criteria from this type of method are strongly linked to the inequalities used. At the beginning, the Jensen inequality has been widely applied to analyze the stability of the DNNs [14–26,33,34,51,52]. In 2013, Seuret et al. [43] presented a Wirtinger-based inequality and proved that it is less conservative than the Jensen inequality. Since then, Wirtinger-based inequality has become the most popular method to estimate the single integral term during the investigation of DNNs [27,37–41,53–56].

Very recently, Zeng et al. proposed a free-matrix-based inequality (FMBI) in [45] and extended it to the research of DNNs [42,50]. To the best of the authors' knowledge, this inequality is the least conservative among the existing inequalities for estimating single integral term. However, there is further room to be investigated using the FMBI. Some slack matrices in the FMBI do not seem to contribute to a reduction of the conservatism. In [42], the FMBI was only used to estimate the single integral term without any augmented vector, while the augmented integral term was still estimated via the Jensen inequality.

It can be expected that stability criteria with less conservatism will be obtained by developing and using a more effective approach to estimate the single integral term. This motivates the present research.

This paper further investigates delay-dependent stability of DNNs following the development of a more effective method to estimate the single integral term (1). The contributions of the paper are summarized as follows:

1. A general free-weighting-matrix (GFWM) approach is developed to estimate single integral term. Based on several zero-value equalities, a new estimation method, named as GFWM approach, is developed by following the basic estimation procedure of the FWM approach. And a new inequality is derived based on the GFWM approach (Lemma 5).
2. Necessary theoretical studies are carried out to compare the GFWM approach and several previous estimation methods. It is proved that the inequality obtained from the GFWM approach can encompass the Wirtinger-based inequality and the FMBI.
3. Several new stability criteria with less conservatism for the DNNs are derived. For generalized neural networks with a time-varying delay, based on two LKFs (one with delay-product-type terms and the other without similar terms), two stability criteria are derived by using the GFWM to estimate the single integral term appearing in the derivative of the LKFs.

The remainder of the paper is organized as follows. Section 2 gives the problem formulation and necessary preliminary. In Section 3, the development of the GFWM approach and its comparison to previous methods are discussed in detail. The GFWM approach is applied to a generalized DNN and several stability criteria are derived in Section 4. In Section 5, three numerical examples are used to demonstrate the benefits of the proposed criteria. Conclusions are given in Section 6.

Download English Version:

<https://daneshyari.com/en/article/4625497>

Download Persian Version:

<https://daneshyari.com/article/4625497>

[Daneshyari.com](https://daneshyari.com)