Applied Mathematics and Computation 294 (2017) 253–263

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Dynamics of a Cournot duopoly game with bounded rationality based on relative profit maximization

A.A. Elsadany*

Department of Basic Science, Faculty of Computers and Informatics, Ismailia 41522, Suez Canal University, Egypt

ARTICLE INFO

Keywords:
Cournot game
Relative profit maximization
Boundedly rational players
Nash Equilibrium point
Bifurcation
Chaos

ABSTRACT

The dynamics of a Cournot duopoly with relative profits maximizations and costs function with externalities is considered. Results concerning the equilibria of the economic model and their stability are presented and the occurrence of bifurcations is stated. A double route to chaotic dynamics, via flip bifurcations and via Neimark–Sacker bifurcations for game is studied. Numerical experiments are presented.

1. Introduction

Many researchers have developed and extensively examined the different variants of oligopoly models. Initially, the existence and uniqueness of the equilibrium of the different types of oligopolies was the main concern. Stability in oligopoly has been studied by Furth [1]. Duopoly is an oligopoly game of two firms competition which have dominant influence and control over a market. In order to maximize the profit, each firm takes action based on the reaction from its rival to compete with its rival. Later the dynamic extensions of these models became the focus. The introduction of complex behavior phenomena in Cournot oligopoly games is well documented in the mathematical economics literature, starting with Rand [2] and Dana and Montrucchio [3]. In fact, the dynamical properties of Cournot duopoly games have been discussed by Puu [4,5] who showed that trajectories may not converge to the Nash equilibrium and that complex trajectories are possible. The local stability of best reply and gradient games with applications to imperfectly competitive models has been examined in [6]. Bischi and Naimzada [7] researched the dynamics of bounded rationality duopoly model. Agiza and Elsadany [8] has studied heterogeneous duopoly games, and in particular they investigated the nonlinear dynamics emerging in these kinds of game. Dubiel-Teleszynski [9] investigated a duopoly game with adjusting heterogeneous players. Sun and Ma [10] discussed the stability of a nonlinear Chinese cold rolled steel market model. Nonlinear oligopolies have been surveyed in [11]. Also, behavioral rationality and heterogeneous expectations in complex economic models has been surveyed by Hommes [12]. Tramontana [13] has concerned the dynamics of a Cournot duopoly with isoelastic demand function which one player has bounded rationality and the other has naive expectation. Fanti and Gori [14] studied the dynamic of properties of a differentiated duopoly with quantity competition. Tramontana and Elsadany [15] and Guirao et al. [16] investigated the dynamics of Cournot oligopoly games when increasing the number of players. Dynamics analysis of monopoly market has been studied in [17]. Complex dynamics and chaos control of heterogeneous duopoly games are analyzed in many other researches [18–21]. Agiza et al. [22] has analyzed the dynamics of a modified Puu duopoly game. Fanti [23] studied the dynamical behaviors of a banking duopoly with capital regulations. Askar [24] introduced a Cournot duopoly game with

* Fax: +20 643320267.
E-mail addresses: aelsadany1@yahoo.com, aelsadany@ci.suez.edu.eg

http://dx.doi.org/10.1016/j.amc.2016.09.018
0096-3003/

concave demand function under impact of cost uncertainty. Elsadany and Matouk [25] considered the effect of delay on stability of Nash equilibrium point in a discrete Cournot duopoly model. Other studies discussed more realistic mechanisms through which firms form their strategies about decisions of competitors, and have shown that the Cournot duopoly game may lead to periodic cycles and deterministic chaos [26–29].

In recent years, more researchers began to study evolutionary game theory [30]. The evolutionary games on multilayer networks is introduced in [31]. Wang and Perc [32] studied the aspiring to the fittest and promotion of cooperation in the prisoner's dilemma game. Wang et al. [33] analyzed the optimal interdependence between networks for the evolution of cooperation. They have deduced that, when cooperation to be optimally promoted, the interdependence should stem only from an intermediate fraction of links connecting the two networks, and that those links should affect the utility of players significantly. Also, they concluded that the interdependence of interaction networks offers several exciting possibilities for further research related to evolutionary games, and it ought to bring the models a step closer to actual conditions, given that networks indeed rarely exist in an isolated state. Effect of heterogeneous sub-populations on the evolution of cooperation has been studied by Huang et al. [34]. Cooperative behavior evolution of small groups on interconnected networks is examined in [35]. For more related papers, readers are advised to have a look at some important works [36–40].

Another branch of literature considered different mechanisms where, the firms seek to maximize the relative profits instead of their absolute profits themselves. The relative profit of one firm is the difference between its absolute profit and the absolute profit of the rival firm. We think that seeking for relative profit or utility is based on the human nature. Even if a person earns big money, if his brother/sister or close friend earns bigger money than him, he is not sufficiently happy. On the other hand, even if he is very poor, if his competitor is poorer, he may be consoled by that fact [41]. In recent years, maximizing relative profit instead of absolute profit has aroused the interest of economists. Recently [42] analyzed a static Cournot duopoly game which is derived from relative profit function.

In this work, we will study dynamic Cournot duopoly game where the duopoly firms use the relative profits instead of their absolute profits themselves which is closer to reality. We will investigate a repeated game model where the boundedly rational firms update their quantity strategies, at discrete time periods, by an adjustment mechanism based on a local estimate of the marginal profit. Sufficient conditions will be determined for the existence of configurations of several asymptotically stable equilibrium points, for given values of parameters. The occurrence of bifurcation phenomena in our game will be investigated. Moreover, some analysis and numerical simulations are carried to show the complex dynamics of the game from bifurcation to chaos. We expect the dynamics will be similar to replicator dynamics in biology [30].

The paper is structured as follows: in Section 2, the model is given. In Section 3, we discuss the dynamical analysis of the game. Also, we derive analytically the stability regions of Cournot–Nash equilibrium point and their bifurcation behaviors. Also, In Section 4 numerical simulations are used to present the bifurcation diagrams, maximum Lyapunov exponents and strange attractors. Finally, the paper is concluded in Section 5.

2. Model

We consider a market for differentiation products with inverse demand function. Each firm i chooses a nonnegative real number q_i, which is the amount of the product to be supplied by her. The strategy profile $\mathbf{q} = (q_1, q_2, \ldots, q_n)$ results in a total supply denoted by $\mathbf{Q} = \sum_{i=1}^{n} q_i$, and a corresponding market price $P_i\,(Q)$. The payoff of ith firm is calculated as follows:

$$\pi_i(q_i, \mathbf{q}_{-i}) = q_i P_i(Q) - C_i(q_i)$$

where q_i is the output of firm i and $C_i(q_i)$ its cost function. We have used the standard notation $\mathbf{q}_{-i}$ to indicate the vector $\mathbf{q}$ with the component q_i omitted. The optimal output quantities of the firms could be determined by solving the following problem

$$\max_{q_i} \pi_i(q_i, \mathbf{q}_{-i}) = q_i P_i(Q) - C_i(q_i)$$

In order to maximize profits for the firm put a partial derivative of $\pi_i(q_i, \mathbf{q}_{-i})$ with respect to q_i equal to zero

$$\frac{\partial \pi_i}{\partial q_i} = P_i + \frac{\partial P_i}{\partial q_i} q_i - \frac{\partial C_i(q_i)}{\partial q_i} = 0$$

and these reaction functions are derived

$$q_i^* = R_i(\mathbf{q}_{-i}) = \arg\max_{q_i} \pi_i(q_i, \mathbf{q}_{-i})$$

For any discrete adjustment process at each time period each firm must form an expectation of the rivals quantities in the subsequent period in order to determine the corresponding profit-maximizing quantity.

Now, we consider model of Cournot duopoly game based on relative profit maximization in the next section. Singh and Vives [43] assumed that the utility function of the representative consumer in the market is given by:

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2bq_1q_2), \tag{1}$$

subject to the budget constraint $p_1q_1 + p_2q_2 + y = M$. The inverse demand functions of the goods produced by the two firms that come from the maximization by the representative consumer of Eq. (1) subject to the budget constraint, are the