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Approximation of Baskakov type Pólya–Durrmeyer operators

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ABSTRACT

In the present paper we propose the Durrmeyer type modification of Baskakov operators based on inverse Pólya–Eggenberger distribution. First we estimate a recurrence relation by using hypergeometric series. We give a global approximation theorem in terms of second order modulus of continuity, a direct approximation theorem by means of the Ditzian–Totik modulus of smoothness and a Voronovskaja type theorem. Some approximation results in weighted space are obtained. Also, we show the rate of convergence of these operators to certain functions by illustrative graphics using the Maple algorithms.

1. Introduction

The classical positive, linear Baskakov operators $(V_n)_{n\in\mathbb{N}}$ were introduced by Baskakov [7] and are defined as follows:

$$V_n(f;x) := \sum_{k=0}^{\infty} f\left(\frac{k}{n}\right)\binom{n+k-1}{k}\frac{x^k}{(1+x)^{n+k}},$$

for every $f \in C_2^*[0,\infty)$ and $x \in [0,\infty)$, where $C_2^*[0,\infty)$ is the weighted space

$$C_2^*[0,\infty) := \left\{ f \in C[0,\infty) : \frac{f(x)}{1+x^2} \text{ is convergent as } x \to \infty \right\},$$

endowed with the norm $\|f\|_* := \sup_{x\geq 0} \frac{|f(x)|}{1+x^2}$.

Lemma 1. *The Baskakov operators satisfy the following properties:*

$$V_n(e_1;x) = x;$$
$$V_n(e_2;x) = \frac{x}{n} + x^2\frac{n+1}{n};$$
$$V_n(e_3;x) = \frac{x}{n^2} + 3x^2\frac{n+1}{n^2} + x^3\frac{(n+1)(n+2)}{n^2};$$
$$V_n(e_4;x) = \frac{x}{n^3} + 7x^2\frac{n+1}{n^3} + 6x^3\frac{(n+1)(n+2)}{n^3} + x^4\frac{(n+1)(n+2)(n+3)}{n^3}.$$

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For more details the reader should consult the book of Gupta and Agarwal [16] and the paper of Becker [8].

The Pólya–Eggenberger distribution was introduced in 1923 (see [15]) and gives the probability of getting k white balls out of n drawings from an urn contains A white and B black balls, if each time one ball is drawn at random and then replaced together with S balls of the same color. The Pólya–Eggenberger distribution (PED) with parameters (n, A, B, S) is defined as follows:

$$P(X=k)=\binom{n}{k}\frac{\prod_{i=0}^{k-1}(A+iS)\prod_{i=0}^{n-k-1}(B+iS)}{\prod_{i=0}^{n-1}(A+B+iS)},\ k=0,1,\dots,n.$$

The inverse Pólya–Eggenberger distribution (IPED) is defined as (see [21]):

$$P(X=k)=\binom{n+k-1}{k}\frac{\prod_{i=0}^{k-1}(A+iS)\prod_{i=0}^{n-1}(B+iS)}{\prod_{i=0}^{n+k-1}(A+B+iS)},\ k=0,1,\dots,n.$$

Based on Pólya–Eggenberger distribution, Stancu considered in [24] a new class of positive linear operators as follows:

$$B_n^{(\alpha)}(f;x)=\sum_{k=0}^{n}p_{n,k}^{(\alpha)}(x)f\left(\frac{k}{n}\right), \tag{1}$$

where $p_{n,k}^{(\alpha)}(x)=\binom{n}{k}\frac{x^{[k,-\alpha]}(1-x)^{[n-k,-\alpha]}}{1^{[n,-\alpha]}}$, α is a non-negative parameter which may depend only on the natural number n and $t^{[n,h]}=t(t-h)(t-2h)\cdots(t-(n-1)h)$, $t^{[0,h]}=1$ represents the factorial power of t with increment h.

In 1970, Stancu [25] considered the modification of Baskakov operators based on inverse Pólya–Eggenberger distribution as follows

$$P_n^{(\alpha)}(f;x)=\sum_{k=0}^{\infty}v_{n,k}^{(\alpha)}(x)f(k/n), \tag{2}$$

where

$$v_{n,k}^{(\alpha)}(x)=\binom{n+k-1}{k}\frac{1^{[n,-\alpha]}x^{[k,-\alpha]}}{(1+x)^{[n+k,-\alpha]}}.$$

In 1989, Razi [23] introduced the Bernstein–Kantorovich operatos based on Pólya–Eggenberger distribution. In [22] was introduced the Bézier variant of genuine-Durrmeyer type operators having Pólya basis functions. Deo et al. [10] considered a Stancu–Kantorovich operators based on inverse Pólya–Eggenberger distribution of the operators (2) and established some direct results. Very recently, Dhamija et al. [13] considered the Stancu–Jain type hybrid operator based on inverse Pólya–Eggenberger distribution and studied approximation properties of these operators which include uniform convergence and degree of approximation. In this direction, significant contribution are given in [1–6,11,17,19].

Very recently, Deo and Dhamija [12] considered new positive linear operators, given by

$$M_n^{(\alpha)}(f;x)=\sum_{k}w_{n,k}^{(\alpha)}(x)f\left(\frac{k}{n}\right), x\in I,\ p\in\mathbb{N}_0=\mathbb{N}\cup\{0\},\ n=1,2,\dots, \tag{3}$$

where

$$w_{n,k}^{(\alpha)}(x)=\frac{n+p}{n+p+(\lambda+1)k}\binom{n+p+(\lambda+1)k}{k}\frac{x^{[k,-\alpha]}(1+\lambda x)^{[n+p+\lambda k,-\alpha]}}{(1+(\lambda+1)x)^{[n+p+(\lambda+1)k,-\alpha]}},$$

and

$$\lambda\in\{-1,0\},\ 0\le\alpha<1,\ I=[0,\infty)\text{ for }\lambda=0\text{ and }I=[0,1]\text{ for }\lambda=-1.$$

The operator (3) is generalized form of above two operators (1) and (2) based on PED and IPED. Furthermore, the Kantorovich variant of the operators (3) was introduced in [14] as follows:

$$L_n^{(\alpha)}(f;x)=(n+p-\lambda)\sum_{k}w_{n,k}^{(\alpha)}(x)\int_{\frac{k}{n+p-\lambda}}^{\frac{k+1}{n+p-\lambda}}f(t)dt,\ x\in I.$$

Inspired by the generalization of Baskakov operators considered by Stancu, we propose the Durrmeyer type modification of the Baskakov operators (2) in the following way:

$$D_n^{(\alpha)}(f;x)=(n-1)\sum_{k=0}^{\infty}v_{n,k}^{(\alpha)}(x)\int_0^{\infty}b_{n,k}(t)f(t)dt, \tag{4}$$

where

$$b_{n,k}(t)=\binom{n+k-1}{k}\frac{t^k}{(1+t)^{n+k}}.$$