



# Finite-time stabilization of switched nonlinear systems with partial unstable modes



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## ABSTRACT

This paper investigates the finite-time stabilization problem of switched nonlinear systems (SNS) in the presence of impulse effects, where both stable subsystems and unstable subsystems coexist. A new notion named mode-dependent average switching frequency (MDASF) is firstly proposed by extending the previous average switching frequency method (ASF). Designing mode-dependent switching law reveals the tradeoff among stable and unstable modes. Based on the estimation on transition matrix and Gronwall–Bellman inequality, mode-dependent feedback controllers are constructed to achieve finite-time stability of the closed-loop systems. Finally, a numerical example is given to verify the efficiency of the proposed method and the validity of our results.

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## 1. Introduction

As a special class of hybrid systems, switched system can be used to model many phenomena such as manufacturing control [1], traffic control [2], chemical processing [3] and communication networks [4], etc. Such a system contains a family of subsystems (both stable subsystems and unstable subsystems) and a logic rule that orchestrates switching among them. SNS is an important class of switched systems, which has a wide application background due to the fact that more and more complex systems need to be presented by nonlinear systems. In the last decade, the study for the stability and stabilization problems of SNS has received growing attention, and many results have been obtained. These results can be divided into three main categories:

- (1) the multiple Lyapunov function (MLF) technique [5–7,33,35,36];
- (2) the (average) dwell time (DT) scheme [8–10];
- (3) the (average) switching frequency method [11–13].

In [11–13], although the stabilization problem of switched systems with a finite set of stable linear subsystems has been solved by the switching frequency method, the case where switched systems contain some unstable modes has not been considered. Besides, it has been shown that, in these literatures, the switching frequency of each subsystem must be smaller than the minimum switching frequency of all possible switching modes, to guarantee the stability of switched systems. Obviously, the minimum switching frequency for all subsystems will give rise to a certain conservatism. Therefore,

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we propose the mode-dependent average switching frequency method (MDASF) which will reduce the restriction of the minimum switching frequency method in this paper.

Generally speaking, when we concern the stability of a system, a distinction between classical Lyapunov asymptotic stability (LAS) and finite-time stability (FTS) should be clearly addressed. LAS deals with the steady-state behavior of the system within infinite time interval, while FTS handles the transient behavior of the system within a finite time interval. To the best of the authors' knowledge, fruitful results on FTS problems have been reported, such as [14–20,29] and the references therein. Du et al. [21] investigated the problem of finite-time boundedness and stabilization for a class of switched linear systems. In [22], the finite-time stability problem for switched nonlinear systems was addressed. Then, in [23], a sufficient condition for the global asymptotic stability of switched nonlinear systems was presented. Recently, in [9,10], the problem of finite-time stability for switched systems in the presence of both non-Lipchitz perturbation and impulse effects was studied, and sufficient conditions for finite-time stability were obtained. Note that, almost all the above papers have not considered the stability analysis of mode-dependent switching. Hence, we will discuss the finite-time stabilization problem of SNS by mode-dependent average switching frequency.

As we know, a switched system with all stable modes may be unstable under inappropriate switching, while a switched system with all unstable modes may be stable under proper switching. This property has been investigated deeply in switched linear systems [24]. However, in some practical applications that modeled by SNS, unstable modes may appear inevitably and can not be avoided. Therefore, the research to the finite-time stabilization of SNS with unstable modes deserves deep investigation due to its academic meaning as well as its practical significance.

Nevertheless, almost all the existing papers are concerned with the stability of the switched linear or nonlinear systems with all stable subsystems, see [25–27] and the references therein. Recently, some results about the stability of switched systems with unstable modes have been reported in [30–32,34]. At present, to the best of our knowledge, no literature is available for the finite-time stabilization issue of SNS with partial unstable modes and impulse effects, where the mode-dependent average switching frequency is used.

Motivated by the above facts, we investigate the finite-time stabilization problem of switched nonlinear systems with impulse effects in this paper. The main contributions of this paper are two folds. The finite-time stabilization of SNS in the presence of unstable modes is studied. Then, by using the mode-dependent average switching frequency (MDASF), which is a new notion that we first proposed, we derive a criterion which ensures the state trajectory remains in a bounded region over a finite-time interval. Such a treatment brings much flexibility and freedom. More specific, we introduce mode-dependent switching law that reveals the tradeoff among stable and unstable subsystems. The paper is organized as follows. In Section 2, we give the definition of the finite-time stabilization and some basic concepts that are useful for discussion. The main results are presented in Section 3, including a mode-dependent switching law and the new MDASF method to derive the finite-time stabilization criterion for the SNS. In Section 4, to show the effectiveness of the obtained results, a numerical example is presented. Section 5 is the conclusion.

The following notations will be used throughout this paper:

- $\mathbb{R}^n$  denotes the  $n$  dimensional Euclidean space;
- $\lambda_{\max}(R)$  and  $\lambda_{\min}(R)$  denote the maximum and minimum eigenvalues of matrix  $R$ ;
- $\|\cdot\|$  refers to the Euclidean vector norm.

## 2. Preliminaries and problem formulation

Consider a class of continuous-time switched nonlinear systems with impulse effects as follows:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) + f_{\sigma(t)}(t, x(t)) + B_{\sigma(t)}u(t), \quad t \in (t_{k-1}, t_k], \quad k = 1, 2, \dots \\ \Delta x(t) &= x(t_k^+) - x(t_k) = F_k x(t), \quad t = t_k \\ x(0^+) &= x_0 \end{aligned} \tag{1}$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  denote the state vector and control input, respectively.  $A_{\sigma(t)}$ ,  $B_{\sigma(t)}$  are constant matrices of appropriate dimensions.  $\sigma(t) : [0, T) \rightarrow \mathfrak{N} = \{1, 2, \dots, \ell\}$  is the switching signal,  $\ell$  is the number of subsystems,  $\{x_0; (i_0, t_0), (i_1, t_1), \dots, (i_k, t_k), \dots, |i_k \in \mathfrak{N}, k = 0, 1, \dots\}$  is the switching sequence, where  $x_0$  is the initial state and  $t_0$  is the initial time.  $\sigma(t) = i_k$  means that the  $i_k$ th subsystem is activated when  $t \in [t_k, t_{k+1})$ . The nonlinear perturbation  $f_i(t, x(t))$  satisfies:

$$\|f_i(t, x(t))\| \leq \gamma \|x(t)\| + \beta(t), \quad i \in \mathfrak{N}$$

where  $\gamma > 0$  is a constant,  $\beta(t)$  is a Lebesgue function satisfying  $\int_0^T e^{\lambda\tau} \beta(\tau) d\tau < \infty$  for some constant  $\lambda > 0$ .  $F_k$  is used to describe the impulse effect at switching instant, and  $F_k$  is a constant matrix. Here, we assume

$$0 \leq \|F_k\| \leq \theta_k$$

For the development of this paper, the following definition, which is called mode-dependent average switching frequency (MDASF), is first defined.

**Definition 1.** For a switching signal  $\sigma(t)$  and any  $T \geq 0$ , let  $N_{\sigma p}(0, T)$  be the switching numbers that the  $p$ th subsystem is activated over the interval  $[0, T]$ , and  $T_p(0, T)$  denote the total running time of the  $p$ th mode over the interval  $[0, T]$ . For

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