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Hedging default risks of CDO tranches in non-homogeneous Markovian contagion models☆

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ABSTRACT

The paper is concerned with the hedging of credit derivatives, in particular synthetic collateralized debt obligations (CDOs) tranches and first to default swap (FTD) with respect to actually traded credit default swaps index (CDS index). In the model, we will relax the name homogeneity assumption, that all the names share the same risk-neutral default. We think of two homogeneous groups of names and the default intensities of each group depending both upon the number of survived names in each subgroup. This results a two dimensional Markov chain setting, since the portfolio state is characterized by the number of survived names in each group. Finally, we have achieved the numerical implementation through trinomial trees, by means of Markov chain techniques. The experimental results show that the new extended hedge model in this paper improves the hedge strategies under the name homogeneity case.

1. Introduction

Recent years have witnessed a surge in demand for credit derivatives, especially with the portfolio credit products like basket default swap (BDS), collateralized debt obligations (CDOs) and index tranches. With exceedingly active market for these portfolio products, a great number of improved models on pricing these portfolios have been published. Among these, factor copula models such as David Li [1], Laurent and Gregory [2], and contagion models such as Jarrow and Yu [3], Yu [4], have a significant impact on the pricing of portfolio credit derivatives. Models begin to provide attention to the hedging issues for CDOs after the market has established the pricing standard, especially during the recent sub-prime mortgage crisis since 2007. Financial institutions trading the credit products such as banks, mutual funds need to hedge increasingly risk in order to lower the uncertainty returns from these products. On the other hand, although the values of highly liquid products are determined by supply and demand, a greater cause for developing the pricing models of these products is that more and more investors need to hedge and arbitrating. That is why hedging of credit derivatives occupies the important position in the studying of credit derivatives. Some scholars such as Cousin and Laurent [5] believed that the next big challenge will aim at developing the models that connect pricing and hedging. Nevertheless, only very few models that consider

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hedging of portfolio credit products exist. Bielecki and Jeanblance [6] establish a representation theorem to derive the hedging portfolio of defaultable contingent claims in a complete market setting and detail the procedure of hedging of FTD with credit default swap (CDS). Frey and Backhaus [7] focus on pricing and hedging of portfolio credit products such as BDS and CDOs in the reduced models with interacting default intensities. The hedging instrument is neither CDS nor CDS index. They choose the set of hedging instruments consists of defaultable zero-coupon bonds issued by some firms in the portfolio and the savings account and solve the self-financing portfolio strategy from a linear system which derived from the theorem of representation of the martingale. However, switching to another hedging instrument such as CDS, could be dealt with more complex and the linear system will become tanglesome form. The hedging of portfolio credit derivatives is also discussed in Frey and Backhaus [8], Rutkowski and Yousiph [9], Scalliet and Jeanblance [10], and Laurent and Cousin [11] The major contribution of our paper is based on the research [11]. In their framework, hedging default risks of CDO tranches with CDS index is the core of their paper. The model assumes all the names share the same risk-neutral default intensity and default intensity of each credit name depends only upon the number of defaults within the pool of assets, and then calculate the hedging strategies through building up a tree. Eventually, a CDO tranche can be perfectly replicated by dynamically trading the CDS index and risk-free asset under the homogeneity of names environment. In this paper, we make an attempt to relax the homogeneity of names assumption and assume that the pool of assets can be divided into two different groups. Each group might correspond to names with identical credit rating or to names from the same industrial sector. The default intensities of names from the same group are identical and depend both on the number of defaults in each subgroup, which leads to a two dimensional Markov chain setting. The calibration methods of loss intensities is provided under the non-homogeneous case. We have achieved numerical implementation through a trinomial tree. Finally, we have established the hedging strategies of a CDO tranche with respect to two CDS index with different maturity.

The remaining sections are organized as follows. Section 2 gives the description of the background of the model and derives the formula of hedging strategies when relaxing the homogeneity assumption. The practical implementations of actual hedging strategies of BDS, such as FTD, and CDO tranches through a trinomial tree have been achieved in next section. Section 4 provides the calibration methods of loss intensities. Some numerical experiments are organized in the Section 5. Further researches and some conclusions are put in the last section.

2. Hedging strategies in non-homogeneous case

2.1. Notation

In the section and in the sequel, we work under the risk -neutral probability $\mathbb{Q}$. We consider a portfolio of n credit references and suppose that the pool of assets can be split into two homogeneous groups with exchangeable risk. The group one includes M_1 equally weighted CDS issuers and M_2 for the group two, where $M_1 + M_2 = n$. We denote by $\tau_i^1, i = 1, \dots, M_1$, $\tau_i^2, i = 1, \dots, M_2$ the default times defined on the probability space $(\Omega, \mathcal{F}, \mathbb{Q})$. Let $N_t^{1i} = I_{\tau_i^1 \le t}, i = 1, \dots, M_1$ and $N_t^{2i} = I_{\tau_i^2 \le t}, i = 1, \dots, M_2$ be the default indicator processes associated of the two groups. Let us recall that the processes $N_t^{1i}, i = 1, \dots, M_1$ are adapted to the global filtration $\mathbb{H}^1 = (\mathcal{H}_t^1, t \ge 0)$ where $\mathcal{H}_t^1 = \bigvee_{i=1}^{M_1} \mathcal{H}_t^{1i}$ and $\mathcal{H}_t^{1i} = \sigma(N_s^{1i}, s \le t)$. Similarly, the processes $N_t^{2i}, i = 1, \dots, M_2$ are adapted to the global filtration $\mathbb{H}^2 = (\mathcal{H}_t^2, t \ge 0)$ where $\mathcal{H}_t^2 = \bigvee_{i=1}^{M_2} \mathcal{H}_t^{2i}$ and $\mathcal{H}_t^{2i} = \sigma(N_s^{2i}, s \le t)$. The filtration $\mathbb{H}$ is defined by $\mathbb{H}^1 \vee \mathbb{H}^2$. Let us denote by $N_t^1 = \sum_{i=1}^{M_1} N_t^{1i}$ and $N_t^2 = \sum_{i=1}^{M_2} N_t^{2i}$ the number of defaults at time t in the entire portfolio respectively in the two groups. In the contagion approach, one starts from a specification of the risk-neutral predefault intensities $\tilde{\alpha}^1, \dots, \tilde{\alpha}^n$. In this paper, the individual predefault intensities only depend upon the number of defaults in each subgroups. Predefault intensities thus take the form $\tilde{\alpha}_t^{1i} = \tilde{\alpha}_1(t, N_t^1, N_t^2), i = 1, \dots, M_1$, $\tilde{\alpha}_t^{2i} = \tilde{\alpha}_2(t, N_t^1, N_t^2), i = 1, \dots, M_2$. For convenience, we assume the recovery rates of the two groups as the constants R_1, R_2. We denote x_1 by the nominal amount for all the names of the group one and x_2 for the group two and they should satisfy the condition $M_1 x_1 + M_2 x_2 = 1$. The aggregate fractional loss of the ith group at time t is given by $L_t^i = x_i(1 - R_i)\frac{N_t^i}{M_i x_i}, i = 1, 2$. As the no simultaneous defaults assumption, and since we are working in the same filtration [5], the intensity of L_t^i or of N_t^i is simply the sum of the individual default intensities. Let us denote by $\lambda_i(t, N_t^1, N_t^2)$ the risk -neutral loss intensity. It is related to the individual predefault intensities by:

$$\lambda_i(t, N_t^1, N_t^2) = (M_i - N_t^i)\tilde{\alpha}_i(t, N_t^1, N_t^2), i = 1, 2. \tag{1}$$

In a Markovian contagion model, the process is more precisely a two dimensional pure birth process since only single default can occur. In fact, the two dimensional process (N_t^1, N_t^2) can be proved to be a continuous time Markov chain on the state space $\{0, \dots, M_1\} \times \{0, \dots, M_2\}$ with loss intensity as in (1).

The generator of the chain $\Lambda(t)$ is as follows: