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Dissipativity based repetitive control for switched stochastic dynamical systems

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ABSTRACT

This paper addresses the issues of dissipativity analysis and repetitive control synthesis for a class of switched stochastic dynamical systems with time-varying delay. By using the lifting technique, the considered one dimensional model is converted into a continuous-discrete stochastic two dimensional delayed model to describe the control and learning actions of the repetitive controller. By employing stochastic system theory together with Lyapunov function technique, a new set of sufficient conditions in terms of linear matrix inequalities (LMIs) is established such that the switched stochastic system in two dimensional delayed model is mean square asymptotically stable and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative. Then, the desired repetitive controller is designed by solving a convex optimization problem established in terms of LMIs. More precisely, repetitive controllers with H_∞, passivity and mixed H_∞ and passivity performances can be obtained as the special cases for the considered system. Finally, numerical examples are provided to demonstrate the effectiveness and potential of the developed design technique.

1. Introduction

Switched systems or hybrid systems are very flexible modeling tool consist of a family of distinct active subsystems subject to a certain switching rule which chooses one of them being active during certain time [12,14,17,18]. Switched systems are used in many practical applications such as in power systems, electrical devices/circuits, manipulator robot, transportation systems and control, communication networks, networked control systems and many other fields [7,8]. Many researchers focused on stability and stabilization of switched systems based on Lyapunov technique, such as the existence of a common Lyapunov function for individual systems guarantees stability of the switched system for arbitrary switching sequences, see [2,13,25,26,36,38,39]. Lin et al. [10] obtained the reliable dissipative control for a family of discrete-time switched singular systems with mixed time delays and multiple actuator failures, where the failure probability of each actuator is individually quantified and is governed by an individual random variable satisfying a certain probabilistic distribution in the interval [0, 1]. A sojourn-probability-dependent method together with Lyapunov technique is proposed to investigate the robust H_∞ control for a class of switched systems with input delays in [24], where the robust mean square stability criteria are obtained for switched systems under the conditions that all sojourn probabilities of the subsystems are known and only partly sojourn probabilities are known. Moreover, the issues of finite-time stability and finite-time stabilization are reported for one dimensional (1D) Ito stochastic systems under the state feedback control scheme in [33,34].

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In particular, for a given periodic reference input, tracking precision is gradually improved by a repetitive controller through the repeated learning actions, where the control input of the previous period is added to the present period to regulate the present control input [11]. As a result, the tracking error can be reduced step by step and finally the output tracks the reference input without steady-state error [20]. Thus, repetitive control systems have attracted great attention and some interesting results on repetitive control systems have been presented (see [5,28,30] and references therein). However, most of the repetitive control design methods are developed for 1D space (the time domain) and focused mainly on the stability issue [19]. On the other hand, a design method based on a two dimensional (2D) continuous-discrete hybrid model for repetitive control is discussed in [37], where 2D system theory is employed and it enables independent adjustment of the control and learning actions. Wu et al. [30] studied the problem of designing a robust repetitive control for a class of linear systems with a relative degree of zero and with time-varying structured uncertainties, where a set of sufficient conditions in terms of LMIs is derived by combining a 2D Lyapunov functional with the structural singular value decomposition of the output matrix to achieve the required result.

On the other hand, it is well known that the dissipative theory gives a framework for the design and analysis of control systems using an input–output description based on energy-related considerations and it serves as a powerful technique in characterizing some important system behaviors such as stability, H_∞ and passivity [1,22,29]. Therefore, the problems based on dissipative theory have become an important issue and received a more attention [6,15,16,23,31]. Wu et al. [27] investigated the problem of dissipativity analysis and synthesis for discrete-time Takagi–Sugeno fuzzy systems with stochastic perturbation and time-varying delay, where fuzzy controller is designed to guarantee the dissipative performance of the closed-loop system with the help of Lyapunov–Krasovskii technique. The problems of stability and dissipativity analysis for static neural networks with interval time-varying delay are studied in [35], where an improved stability criterion is derived for the considered neural networks by employing Wirtinger-based inequality and Lyapunov technique. A new set of sufficient conditions in terms of LMIs for dissipativity analysis of Markov jump systems with polytopic uncertainties and sawtooth delays is derived via feedback sampled-data control in [21]. Cui et al. [3] studied the delay-dependent dissipativity analysis for a general class of singular systems with Markovian jump parameters and mode-dependent mixed time delays by using a novel Lyapunov–Krasovskii functional and stochastic analysis technology. The problem of static output-feedback dissipative control is discussed for a class of linear continuous-time systems based on an augmented system approach in [4], where a necessary and sufficient condition for the existence of a desired controller is derived in terms of LMIs.

To the best of the author's knowledge, the problem of dissipative analysis for stochastic switched dynamical systems with time-varying delay via repetitive control strategy has not been studied yet, which motivates the present work. In order to fill this gap, in this paper, we consider the dissipativity problem for stochastic switched systems with time-varying delay under repetitive controller. The main contributions of this paper are summarized as follows:

(i) Based on the lifting technique, a continuous-discrete stochastic 2D delayed model is proposed to describe the characteristics of a 1D switched stochastic repetitive control system.
(ii) By constructing a proper Lyapunov–Krasovskii functional and using Ito differential rule with stochastic analysis theory, a new set of sufficient conditions is obtained in terms of LMIs such that the formulated stochastic 2D delayed model is mean square asymptotically stable and $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative.
(iii) Based on the obtained LMI conditions, the state feedback gains of the proposed controller are determined. In particular, the gain parameters of the 2D control law can be used directly to adjust the control and learning actions. More precisely, the proposed dissipative repetitive control unifies the problems of H_∞, passivity and mixed H_∞ and passivity control designs in a single framework.
(iv) Finally, numerical examples are provided to illustrate the effectiveness of the proposed repetitive control design.

Notations: The superscript 'T' denotes the transpose, and the notation $X \geq Y (X > Y)$ means that matrix X−Y is positive semi-definite (positive definite, respectively). $\mathbb{R}$ denotes the space of real numbers. $\mathbb{R}^+$ is the set of non-negative real numbers. $\mathbb{Z}^+$ is the set of non-negative integers. $\mathbb{R}^n$ denotes the n dimensional real vector space. I and 0 represent the identity matrix and zero matrix, respectively. $diag\{a_i\}$ denotes a diagonal matrix with the diagonal elements a_i $i = 1, 2, \ldots, n$. The asterisk $*$ in a matrix is used to denote the term that is induced by symmetry. $\mathcal{L}$ is the infinitesimal generator, $\|.\|$ denotes the Euclidean norm. $L_2[0, \infty)$ is the space of square integrable vector valued functions on $[0, \infty)$. $\mathbb{C}^p$ is the p-dimensional vector space over complex numbers. Ξ is the linear space of all the functions from $[0, T]$ to $\mathbb{C}^p$. $\mathbb{L}_2(\mathbb{R}^+, \mathbb{C}^p)$ is the linear space of square integrable functions from $\mathbb{R}^+$ to $\mathbb{C}^p$, and $l_2(\mathbb{Z}^+, \Xi)$ is the linear space of all the functions from $\mathbb{Z}^+$ to Ξ.

2. Problem formulation and preliminaries

In this paper, we consider a switched stochastic dynamical system with time delay in the following form:

$$\begin{cases} dx(t) = \big[\hat{A}_{\sigma(t)}x(t) + \hat{A}_{d,\sigma(t)}x(t-d(t)) + B_{\sigma(t)}u(t) + B_{h,\sigma(t)}h(t)\big]dt + G_{\sigma(t)}x(t)dw(t), \\ y(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}u(t), \\ x(t) = \varphi(t), \quad t \in [-\bar{d}, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}$ are the state and the control input, respectively; $h(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $L_2[0, \infty)$ and $y(t) \in \mathbb{R}$ is the output vector; $\varphi(t) \in \mathbb{R}^n$ is the initial function and $w(t)$ is a 1D zero-mean Wiener