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A coupled level-set and volume-of-fluid method for simulating axi-symmetric incompressible two-phase flows

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ABSTRACT

A coupled level-set and volume-of-fluid (CLSVOF) method, combining the advantages of LS method and VOF method, is presented for simulating axi-symmetric incompressible liquid–gas flow problems. In this method, the interface is implicitly captured by the LS function, and the VOF function is used as a complement to remedy the mass conservation problem aroused in the advection and re-initialization of LS function. We propose a novel explicit algebraic relation between the LS function and VOF function to achieve the coupling of the two methods. Four typical axi-symmetric liquid–gas flow problems, including an equilibrium spherical liquid drop suspending in a quiescent gas environment, a liquid drop falling under the action of gravitational force, a single spherical gas bubble rising in a container filled with quiescent liquid, and the Rayleigh–Taylor instability aroused in a cylindrical container are simulated to validate the CLSVOF method. It is shown that the calculation results obtained by the CLSVOF method agree well with the theoretical solutions and experimental results.

1. Introduction

Numerical calculations of incompressible two-phase flows play essential roles in a wide range of industrial applications and academic researches, *e.g.*, atomization of a liquid jet emanating from a nozzle, splash of a liquid drop falling into a liquid tank and cavitation formation around a pump impeller. The key part in such calculations is to accurately predict the motion of the liquid–gas interface, which can be achieved by various numerical methods. According to the strategy adopted to locate the interface, these numerical methods can be generally categorized as interface tracking methods and interface capturing methods [1].

A typical interface tracking method is so-called "front-tracking method" [2–4], in which the interface is tracked in an adaptive grid while the continuity and momentum equations are solved in a stationary, fixed background grid. The interface jump conditions, such as those for the fluid density, fluid viscosity and pressure, are computed in the interface grid and then transferred to the fixed grid by distributing them within an artificial interface region of a finite thickness. The interface front is advected in a Lagrangian fashion by the velocity interpolated from the flow field solved in the fixed grid. Due to the deterioration after front advection, the interface grid requires adaptation to improve its quality. Another interface tracking method usually used is "surface-marker method" [5–7], in which the phase interface is straightforwardly tracked by a collection of marker particles placed onto it. Similar to the "front-tracking method", these marker particles are advected in a Lagrangian fashion in the flow field solved from Navier–Stokes equations inside a fixed-grid region. This type of methods is particularly suitable for the study of the motion of small amplitude waves and weakly deformed bubbles [8], but it becomes

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difficult to evaluate the interface grid or surface particles when the topology of interface changes. This difficulty limits its application to the simulation involving interface merging and breakup [1].

As another approach to describe the interface, the interface capturing methods have been widely used for direct numerical simulations of free-surface and interfacial flows. In this type of methods, the phase interface is implicitly captured by an additional scalar. The most popular scalar functions adopted to represent the interface are the volume-of-fluid (VOF) function, F, and the level-set (LS) function, ϕ. Accordingly, the VOF method and LS method are proposed to simulate the interfacial dynamics between different phases.

In the VOF method, VOF function (F) is defined as the volume fraction of a certain fluid in each computational cell. $F = 1$ for the cell occupied by the concerning fluid; $F = 0$ for the cell occupied by the other one; $0 < F < 1$ for the cell occupied by both fluids (containing the interface). Some critical implements in the VOF method should be treated carefully. The first one is the numerical scheme for the advection of F. Due to the discontinuous nature of F, the traditional numerical schemes for solving the advection equation of continuous quantities, such as the classic upwind scheme, are no longer suitable for the advection of F. The advection equation of F is usually solved with a geometric-based scheme, in which the interface in each interface cell needs reconstruction according to the distribution of F-values. The VOF methods are classified by the algorithm to reconstruct the interface. In the original VOF method, Hirt and Nichols [9] used a simple line interface calculation (SLIC) method to reconstruct the interface in the interface cell. In such SLIC method, which is only first-order accurate, the reconstructed interface is one or two lines aligned with the cell edges. To improve the accuracy of reconstruction, a linear approximation to the interface, which is usually referred to as the piecewise-linear interface calculation (PLIC) method, has been proposed [10–14]. In the implementation of the VOF method, the surface tension force is usually transformed to be a body force distributed within the interface region of a finite thickness by the continuum surface force (CSF) model [15]. In this model, the interface normal and its derivatives require adequate approximation to accurately evaluate the interfacial curvature and further the surface tension force. The interface normal is computed from the gradient of the F, which must be done carefully in the VOF method because of the discontinuous nature of F. Two types of algorithms are generally used to estimate the interface normal. One type is to compute the derivative of F from the discrete F-values at the cells of a wide and symmetric stencil, such as the center of mass method [16], the central difference method [14], Parker and Youngs' method [17], least squares VOF interface reconstruction algorithm (LVIRA) [18] and efficient least squares VOF interface reconstruction algorithm (ELVIRA) [19]. The other type to compute the derivative of F is to smooth F within the interface region by convolving F with some kernel function, such as a B-spline [15] and a radially symmetric kernel K_8 [20].

Another interface capturing method to simulate the interfacial flow is the LS method [21–24], in which the LS function ϕ is defined as the signed normal distance to the interface. According to this definition, the interface is represented by the zero level-set iso-face $\phi = 0$. The interface is advected implicitly by solving the advection equation of ϕ, which can be solved algebraically with high order discretization schemes due to the continuous nature of ϕ [25,26]. For the same reason, it is easy to calculate the derivatives of ϕ, and subsequently the normal and curvature of the interface. In the LS method, it is essential to keep ϕ a distance function (*i.e.*, $|\nabla\phi| = 1$ holds according to the definition of ϕ) at all calculation instants to estimate the interface curvature accurately [23,26,27]. However, this condition cannot be guaranteed after advection of ϕ [23,28]. Therefore, a "re-initialization" process is required to maintain ϕ as a distance function [23,24,29]. Although the continuous nature of ϕ makes the LS method a powerful technique in simulating the interfacial flow, the main drawback is that the fluid mass is not conserved strictly because of the numerical dissipation in discretizing advection equation and the erroneous movement of the zero level-set in the re-initialization process. Several efforts have been made towards remedying the problem on mass-conservation in the LS method, such as discretizing advection equation with higher order schemes [30,31], using adaptive mesh near the interface [32,33] and developing an interface-preserving re-initialization algorithm [26,34].

From aforementioned characteristics of the VOF and LS methods, it can be seen that solely using either one is not accurate enough. The VOF method is inherently mass-conserving, but its poor behavior in calculating curvature due to its discontinuous property limits its application. The LS method has advantages in estimating curvature due to the continuity nature of ϕ; however, it usually arouses mass loss/gain in the advection and re-initialization processes. The advantages and disadvantages of the two methods are complementary, so the coupled level-set and volume-of-fluid (CLSVOF) methods, taking advantages of both methods, have been proposed recently to simulate the two-phase flow [27,35–43]. The major difference between these CLSVOF methods is the manner of coupling between VOF function and LS function. In the original CLSVOF method proposed by Sussman and Puckett [40], the piecewise-linear interface requires to be reconstructed for assigning the level set function with the exact signed normal distance to the interface, which is quite complicated to implement. Sun and Tao [39] developed a simpler CLSVOF method, in which only the VOF function F needs to be advected and ϕ is calculated by a simple iterative geometric operation. Van der Pijl et al. [27,41] proposed an algebraic relation between VOF function and LS function for two-dimensional (2D) and three-dimensional (3D) configurations, which significantly simplifies the coupling implementation. Recently, Luo et al. **[**36**]** adopted this relation to develop a CLSVOF method for 3D direct numerical simulation of liquid atomization.

For the axi-symmetric phenomena, such as a bubble rising in a quiescent liquid due to buoyancy [3] and a liquid drop splashing on a liquid layer [44], it is more efficient to discretize the governing equations on the 2D cylindrical coordinate system than 3D Cartesian coordinate system because the cost of computation time of the former case is significantly diminished. Although Sussman and Puckett [40] have developed a CLSVOF algorithm for axi-symmetric two-phase flows, the