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Convexity and monotonicity properties of the local integro cubic spline

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ABSTRACT

In this paper, we consider shape-preserving properties of the local integro-cubic spline. The sufficient conditions for convex, monotone, or positive approximation are given. Some examples are provided to illustrate shape-preserving properties of the splines.

1. Introduction

In practice, the shape-preserving properties of splines have attracted interest of many authors, (see, for example, [2–13]). The sufficient conditions for monotonicity and convexity were obtained by Miroshnichenko for cubic spline of class C^2 in [7] and, by Kvasov and Schumaker for the weighted quadratic spline in [6,10] and the cubic splines in Kvasov [5], respectively. In the last decade, besides of interpolatory splines, integro-splines [1,15,17] have been attracting more interest. Recently, Boujraf et al. [2] proposed a simple method for constructing integro spline quasi-interpolants. They did not investigate shape preserving properties of the integro spline quasi-interpolant.

The aim of this paper is to investigate the monotonicity and convexity properties of the local integro cubic spline given by Zhanlav and Mijiddorj in [15]. The paper is organized as follows. Section 2 is devoted to discovering some useful properties of the local integro cubic spline (LICS for short). The sufficient convexity and monotonicity conditions for LICS are given in Section 3. The sufficient comotonicity conditions for LICS are given in Section 4. The results of numerical experiments are presented in Section 6.

2. Some useful properties of LICS

We introduce uniform partition on $[a, b]$, $x_i = a + ih$ for $i = 0, 1, \dots, k$ with $h = (b - a)/k$. Let $S(x)$ be a local integro cubic spline belonging to $C^2[a, b]$ and satisfying the following conditions [1,15]

$$\int_{x_{i-1}}^{x_i} S(x)dx = \int_{x_{i-1}}^{x_i} u(x)dx = I_i, \quad i = 1(1)k. \tag{2.1}$$

We will use the B-representation of $S(x)$

$$S(x) = \sum_{j=-1}^{k+1} \alpha_j B_j(x), \tag{2.2}$$

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where $B_j(x)$ are the normalized cubic B-splines that form a basis for space of C^2 cubic splines. According to the properties of B-splines we have [12]

$$S_i = \frac{\alpha_{i+1} + 4\alpha_i + \alpha_{i-1}}{6}, \tag{2.3a}$$

$$m_i = \frac{\alpha_{i+1} - \alpha_{i-1}}{2h}, \quad i = 0, 1, \dots, k, \tag{2.3b}$$

$$M_i = \frac{\alpha_{i+1} - 2\alpha_i + \alpha_{i-1}}{h^2}. \tag{2.3c}$$

For the integro cubic spline (2.2) satisfying the conditions (2.1) hold the following relations [1,15]

$$\alpha_{i-2} + 12\alpha_{i-1} + 22\alpha_i + 12\alpha_{i+1} + \alpha_{i+2} = \frac{24}{h}(I_i + I_{i+1}), \quad i = 1(1)k-1. \tag{2.4}$$

In [15] were obtained the explicit and approximate formulae

$$\alpha_i = \frac{1}{6h}(-I_{i-1} + 4I_i + 4I_{i+1} - I_{i+2}), \quad i = 2(1)k-2, \tag{2.5}$$

and

$$\alpha_{i-1} + \alpha_i + \alpha_{i+1} = \frac{3}{2h}(I_i + I_{i+1}), \quad i = 1(1)k-1, \tag{2.6}$$

with accuracy $O(h^4)$. The remainder coefficients in (2.2) were determined from (2.4) to (2.6) explicitly and we present the final results

$$\begin{aligned} \alpha_{-1} &= \frac{1}{6h}(35I_1 - 59I_2 + 40I_3 - 10I_4), \\ \alpha_i &= \frac{1}{6h}(10I_{i+1} - 5I_{i+2} + I_{i+3}), \quad i = 0, 1, \end{aligned} \tag{2.7}$$

and

$$\begin{aligned} \alpha_i &= \frac{1}{6h}(10I_i - 5I_{i-1} + I_{i-2}), \quad i = k-1, k, \\ \alpha_{k+1} &= \frac{1}{6h}(35I_k - 59I_{k-1} + 40I_{k-2} - 10I_{k-3}), \end{aligned} \tag{2.8}$$

in which we used the following relation

$$I_{i-2} - 4I_{i-1} + 6I_i - 4I_{i+1} + I_{i+2} = h^5 u_i^{(4)} + O(h^6), \tag{2.9}$$

where $u \in C^5$. Thus, all the coefficients in (2.2) were determined completely by the explicit formulae (2.5), (2.7) and (2.8). As a consequence, for LICS (2.2) with coefficients given by (2.5), (2.7) and (2.8) we have

$$S'''_{i+0} = S'''_{i-0}, \quad i = 1, 2, \quad k-2, k-1. \tag{2.10}$$

The conditions (2.10) for $i = 1$ and $i = k-1$ were known as not-a-knot ones [1,4,12]. The conditions (2.10) can be rewritten in terms of M_i as:

$$M_{i+1} - 2M_i + M_{i-1} = 0, \quad i = 1, 2, \quad k-2, k-1. \tag{2.11}$$

The conditions (2.10) or (2.11) play the role of end conditions for the local integro cubic spline. It was pointed out that in [1,15,17] were proposed different kinds of end conditions in order to determine the integro cubic spline uniquely. In Zhanlav and Mijiddorj [15] LICS was constructed, in which the conditions (2.10) were used in a implicit way. In connection with this it is correct to say that LICS satisfying the end conditions (2.10) exists unique and its coefficients are determined by explicit formulae (2.5), (2.7) and (2.8). In our opinion, the conditions (2.10) are more compact and useful in contrast to the others [1,15,16], because we have no information about the functions and its derivative values at knots.

The following consistency relations [1] hold

$$m_{i-1} + 10m_i + m_{i+1} = \frac{12}{h^2}(I_{i+1} - I_i), \quad i = 1(1)k-1. \tag{2.12}$$

Now we show that the values of S_i, m_i and M_i can be locally approximated in terms of I_j. Indeed, if we take into account the formulae (2.5)–(2.9), and (2.12), then from (2.3) we obtain

$$\begin{aligned} S_0 &= \frac{1}{12h}(25I_1 - 23I_2 + 13I_3 - 3I_4), \\ S_1 &= \frac{1}{12h}(3I_1 + 13I_2 - 5I_3 + I_4), \end{aligned}$$