Applied Mathematics and Computation 293 (2017) 244–253

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Mixed H_∞ and passive control for singular systems with time delay via static output feedback☆

Jian Chen[a,b], Chong Lin[a,*], Bing Chen[a], Qing-Guo Wang[c]

[a] Institute of Complexity Science, Qingdao University, Qingdao 266071, China
[b] College of Automation Engineering, Qingdao University of Technology, Qingdao 266555, China
[c] School of Electrical Engineering, University of Johannesburg, Aucklandpark 2006, Johannesburg, South Africa

ARTICLE INFO

Keywords:
Singular system
Time delay
Mixed H_∞ and passive control
Static output feedback

ABSTRACT

In this paper, the mixed H_∞ and passive control problem is studied for singular systems with time delay. A sufficient condition is proposed to ensure the considered system to be regular, impulse-free and stable with mixed H_∞ and passivity performance level. Then the criterion is adopted to design a static output feedback controller in terms of bilinear and linear matrix inequalities. An algorithm is established to cast the nonlinear feasibility problem into a sequential minimization problem. Finally, a numerical example is presented to show the effectiveness of the proposed methods.

1. Introduction

In the past decade, singular systems have been extensively studied due to the fact that they can better describe physical systems than regular ones. Moreover, time delays often exist in many dynamical systems. The existence of time delay frequently causes poor performance and instability. So studies of the stability analysis and stabilization for singular time-delay systems are of theoretical and practical importance. Accordingly, both delay-independent [1,2] and delay-dependent [3–5,22] stability conditions for singular time-delay systems are derived. The problems of stabilization are investigated in [1,6,7]. The filtering and H_∞ filtering problems for singular time-delay systems are discussed in [8–11].

On the other hand, the problems of H_∞ control are widely investigated for all sorts of systems with the development of a linear matrix inequality (LMI) approach [10,12,21,25]. The H_∞ controllers are designed for singular time-delay systems in [12]. The dissipativity analysis and design are studied in [13,14,23] for continuous- and discrete-time delay systems. The problems of mixed H_∞ and passive filtering are discussed in [11,15] for continuous and discrete time-delay singular systems. But very little attention is paid to the mixed H_∞ and passive control problems for time-delay singular systems.

In control theory, state variables are not always available for feedback in many control systems. In this case, the design of static output feedback (SOF) controller is an important research topic and has found many practical applications. But the SOF controller design criteria for singular systems with time delays cannot be expressed in terms of LMIs. The conditions in [16,24,30,31] are characterized by LMIs, which are tractable and reliable in numerical computations. However, these LMI methods are often conservative. An LMI-based iterative method is proposed in [17] based on the method in [18,19]. The cone-complement linearization method in [29] cannot be extended to deal with the SOF control design problems for singular

☆ This work is supported by the National Natural Science Foundation of China (61673227, 61473160, 61573204, 61503222).
* Corresponding author.
E-mail addresses: janec@163.com (J. Chen), linchong_2004@hotmail.com (C. Lin), chenbing1958@126.com (B. Chen), wangq@uj.ac.za (Q.-G. Wang).

http://dx.doi.org/10.1016/j.amc.2016.08.029
0096-3003/

systems. The improved path-following method [28] is mainly for BMI problems, but it depends on the initial values. To the best of our knowledge, the delay-dependent control problems for singular systems have not been fully investigated. There has room for improvement.

In this paper, we study the mixed H_∞ and passive control problems for singular time delay systems via static output feedback. Based on the auxiliary function-based integral inequality, a new sufficient condition is proposed to ensure the considered system to be admissible with mixed H_∞ and passivity performance level. Then the criterion is adopted to design a static output feedback controller in terms of bilinear and linear matrix inequalities, and a new algorithm is established to solve these matrix inequalities.

This paper is organized as follows. Section 2 is the problem formulation and preliminaries. Section 3 gives the stability and stabilization criteria and the method of SOF controller design. Section 4 provides a numerical example to show the merits and efficiency of the results. Section 5 concludes this paper.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \le Y$ (respectively, $X < Y$) means that the matrix $X - Y$ is negative semidefinite (respectively, negative definite). I and 0, respectively, are the identity matrix and zero matrix with appropriate dimensions. The superscripts -1 and T denote the matrix inverse and transpose, respectively. The symbol $*$ is used to denote a matrix which can be inferred by symmetry. For any $A \in \mathbb{R}^{n\times n}$, we define $Sym\{A\} = A + A^T$. $L_2[0, \infty)$ denotes the space of square integrable functions on $[0, \infty)$, and $\lambda_{\min}(\cdot)$ denotes the smallest eigenvalue of a given matrix. Let $M^\perp$ be the orthogonal complement of M which is defined as the matrix with maximum column rank satisfying $MM^\perp = 0$ and $M^{\perp T}M^\perp > 0$.

2. Problem formulation and preliminaries

Consider the following singular time-delay system:

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) + B_\omega \omega(t), \\ y(t) = Cx(t) + C_d x(t-d), \\ z(t) = C_z x(t), \\ x(t) = \phi(t), \ \ t \in [-d, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $\omega(t) \in \mathbb{R}^p$ is the disturbance input vector that belongs to $\mathcal{L}_2[0, \infty)$, $y(t) \in \mathbb{R}^s$ is the measured output, $z(t) \in \mathbb{R}^q$ is the signal to be estimated, $\phi(t)$ is an initial condition, and the time delay d is a positive integer. The matrix $E \in \mathbb{R}^n$ may be singular and it is assumed that $rankE = r \le n$. A, A_d, B, B_ω, C, C_d, and C_z are real constant matrices with appropriate dimensions. We assume that B is full column rank.

Before moving on, the following definitions and lemmas are required.

Definition 1 ([1]). (i) Singular system

$$E\dot{x}(t) = Ax(t) + A_d x(t-d) \tag{2}$$

is said to be regular and impulse free, if the pair (E, A) is regular and impulse free.

(ii) System (2) is said to be stable if for any scalar $\varepsilon > 0$, there exists a scalar $\delta(\varepsilon) > 0$ such that, for any compatible initial conditions $\phi(k)$ satisfying $sup_{-d\le t\le 0} \parallel \phi(t) \parallel \le \delta(\varepsilon)$, the solution $x(t)$ of system (2) satisfies $\|x(t)\| \le \varepsilon$ for any $t \ge 0$, moreover $lim_{t\to\infty} x(t) = 0$.

(iii) System (2) is said to be admissible if it is regular, impulse free and stable.

Definition 2. System (1) is said to have mixed H_∞ and passivity performance γ, if under zero initial condition, there exists a scalar $\gamma > 0$ such that

$$\int_0^{t^*} (-\gamma^{-1}\theta z^T(\alpha)z(\alpha) + sym\{(1-\theta)z^T(\alpha)\omega(\alpha)\})d\alpha \ge -\gamma \int_0^{t^*} \omega^T(\alpha)\omega(\alpha)d\alpha \tag{3}$$

for all $t^* > 0$ and any non-zero $\omega(t) \in \mathcal{L}_2[0, \infty)$, where $\theta \in [0, 1]$ represents a weighting parameter that defines the trade-off between H_∞ and passivity performance.

Lemma 1 ([20]). *For a positive definite matrix $R \in \mathbb{R}^{n\times n}$, and a differentiable function $\{x(u)|u \in [a, b]\}$, the following inequality holds:*

$$\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \ge \frac{1}{b-a}\Omega_1^T R\Omega_1 + \frac{3}{b-a}\Omega_2^T R\Omega_2 + \frac{5}{b-a}\Omega_3^T R\Omega_3, \tag{4}$$

where

$$\Omega_1 = x(b) - x(a),$$

$$\Omega_2 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(s)ds,$$

$$\Omega_3 = x(b) - x(a) + \frac{6}{b-a}\int_a^b x(s)ds - \frac{12}{(b-a)^2}\int_a^b \int_\beta^b x(s)dsd\beta.$$