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Solving system of linear Stratonovich Volterra integral equations via modification of hat functions

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A B S T R A C T

This paper proposes an efficient method for solving system of linear Stratonovich Volterra integral equations. Stochastic operational matrix of modification of hat functions (MHFs) is determined. By using MHFs and their stochastic operational matrix of integration, a system of linear Stratonovich Volterra integral equations can be reduced to a linear system of algebraic equations. Thus we can solve the problem by direct methods. Also, we prove that the rate of convergence is $O(h^3)$. Efficiency of this method and good degree of accuracy are confirmed by numerical examples.

1. Introduction

The Stratonovich integral equations developed simultaneously by Stratonovich [1] and Fisk [2]. In stochastic processes, the Stratonovich integral is a stochastic integral that is the most common alternative to the Itô integral. Although the Itô integral is the usual choice in applied mathematics, the Stratonovich integral is frequently used in physics. In physics, stochastic integrals occur as the solutions of Langevin equations. A Langevin equation is a coarse-grained version of a more microscopic model, depending on the problem in consideration, Stratonovich or Itô interpretation or even more exotic interpretations such as the isothermal interpretation.

The Wong–Zakai theorem [3] states the physical systems with non-white noise spectrum characterized by a finite noise correlation time t that can be approximated by a Langevin equations with white noise in Stratonovich interpretation in the limit where t tends to zero. Because the Stratonovich calculus satisfies the ordinary chain rule, stochastic differential equations (SDEs) in the Stratonovich sense can be meaningfully defined on arbitrary differentiable manifolds, rather than just on R^n. This is not possible in the Itô calculus, since here the choice of coordinate system would affect the SDEs solution.

In some circumstances, integrals in the Stratonovich definition are easier to manipulate. Unlike the Itô calculus, Stratonovich integrals are defined such that the chain rule of ordinary calculus holds. Perhaps the most common situation in which these are encountered is solving the Stratonovich SDEs. These are equivalent to Itô SDEs and it is possible to convert to each other.

The Stratonovich integral can be defined in a manner similar to the Riemann integral, that is as a limit of Riemann sums. Stochastic integrals can rarely be solved in analytic form that making stochastic numerical integration an important topic in all uses of stochastic integrals. Various numerical approximations converge to the Stratonovich integral, and are used to solve Stratonovich SDEs [4–6]. Numerical schemes to Stratonovich equations have been well developed [1–18]. However,

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there are still very few papers discussing the numerical solutions for Stratonovich Volterra integral equations in comparison to many papers about deterministic integral equations [19–22].

In this paper, we use MHFs to solve the system of linear Stochastic Volterra integral equation as follows

$$\mathbf{f}(x) = \mathbf{g}(x) + \int_0^x \mathbf{k1}(x,y)\mathbf{f}(y)dy + \int_0^x \mathbf{k2}(x,y)\mathbf{f}(y) \circ dB_y, \quad x \in D = [0,T], \tag{1}$$

where

$$\mathbf{f}(x) = [f_1(x), f_2(x), \ldots, f_n(x)]^T, \tag{2}$$

$$\mathbf{g}(x) = [g_1(x), g_2(x), \ldots, g_n(x)]^T, \tag{3}$$

$$\mathbf{k1}(x,y) = \left[k1_{ij}(x,y)\right], \quad i,j = 1,2,\ldots,n, \tag{4}$$

$$\mathbf{k2}(x,y) = \left[k2_{ij}(x,y)\right], \quad i,j = 1,2,\ldots,n, \tag{5}$$

$g_i(x) \in \mathcal{X} = C^3(D)$ and $k1_{ij}(x,y), k2_{i,j}(x,y) \in \mathcal{X} \times \mathcal{X}$ are known functions and $f_i(x) \in \mathcal{X}$ is unknown functions, for $i,j = 1,2,\ldots,n$. Note that the symbol $\circ$ between integrand and the stochastic differential is used to indicate Stratonovich integrals.

2. MHFs and their properties

2.1. Definitions of MHFs

The family of first $(m+1)$ MHFs is defined as follows [23]

$$h_0(x) = \begin{cases} \dfrac{1}{2h^2}(x-h)(x-2h) & 0 \leqslant x \leqslant 2h, \\ 0 & \text{otherwise}, \end{cases}$$

if i is odd and $1 \leqslant i \leqslant m-1$,

$$h_i(x) = \begin{cases} \dfrac{-1}{h^2}(x-(i-1)h)(x-(i+1)h) & (i-1)h \leqslant x \leqslant (i+1)h, \\ 0 & \text{otherwise}, \end{cases}$$

if i is even and $2 \leqslant i \leqslant m-2$,

$$h_i(x) = \begin{cases} \dfrac{1}{2h^2}(x-(i-1)h)(x-(i-2)h) & (i-2)h \leqslant x \leqslant ih, \\ \dfrac{1}{2h^2}(x-(i+1)h)(x-(i+2)h) & ih \leqslant x \leqslant (i+2)h, \\ 0 & \text{otherwise}, \end{cases}$$

and

$$h_m(x) = \begin{cases} \dfrac{1}{2h^2}(x-(T-h))(x-(T-2h)) & T-2h \leqslant x \leqslant T, \\ 0 & \text{otherwise}, \end{cases}$$

where $m \geq 2$ is an even integer and $h = \frac{T}{m}$. According to definition of MHFs, we have

$$h_i(jh) = \begin{cases} 1 & i = j, \\ 0 & i \neq j, \end{cases}$$

$$h_i(x)h_j(x) = \begin{cases} 0 & i \text{ is even and } |i-j| \geqslant 3, \\ 0 & i \text{ is odd and } |i-j| \geqslant 2, \end{cases}$$

and

$$\sum_{i=0}^{m} h_i(x) = 1.$$

Let us write the MHFs vector $H(x)$ as follows

$$H(x) = [h_0(x), h_1(x), \ldots, h_m(x)]^T; \; x \in D. \tag{6}$$