Applied Mathematics and Computation 293 (2017) 265–286

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Non-iterative regularized MFS solution of inverse boundary value problems in linear elasticity: A numerical study

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ARTICLE INFO

Keywords:
Linear elasticity
Inverse boundary value problem
Method of fundamental solutions (MFS)
Singular value decomposition (SVD)
Regularization

ABSTRACT

The numerical reconstruction of the missing Dirichlet and Neumann data on an inaccessible part of the boundary in the case of two- and three-dimensional linear isotropic elastic materials from the knowledge of over-prescribed noisy measurements taken on the remaining accessible boundary part is investigated. This inverse problem is solved using the method of fundamental solutions (MFS), whilst its stabilization is achieved through several singular value decomposition (SVD)-based regularization methods, such as the Tikhonov regularization method [48], the damped SVD and the truncated SVD [18]. The regularization parameter is selected according to the discrepancy principle [40], generalized cross-validation criterion [14] and Hansen's L-curve method [20].

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1. Introduction

The method of fundamental solutions (MFS) was originally proposed in the early 1960s by Kupradze and Aleksidze [25] and since its introduction as a numerical method by Mathon and Johnston [39], it has been successfully applied to a large variety of direct and inverse problems in science and engineering, see e.g. the survey papers by Fairweather and Karageorghis [10], Golberg and Chen [13], Fairweather et al. [11], and Karageorghis et al. [23]. The MFS is a meshless boundary collocation method which belongs to the family of so-called Trefftz methods [24,49], may be regarded as an approximation of the indirect boundary element method (BEM) [27,46] and is applicable to boundary value problems for which a fundamental solution of the operator in the governing equation is known. In spite of the aforementioned restriction, the MFS has become very popular primarily because of the ease with which it can be implemented, in particular for the solution of problems in complex geometries, as well as its low computational cost. The MFS with fixed singularities has been applied to obtain the numerical solution of several direct problems in elasticity, such as two-dimensional [7,8,29,43], axisymmetric [21,44] and three-dimensional problems [42,45]. Moreover, it is important to mention that numerous studies have been devoted very recently to the application of the MFS to the numerical solution of various problems in science and engineering, such as direct problems in uncoupled and coupled magnetoencephalography and electroencephalography [1,2], vibrations induced by the underground railway traffic [4], direct problems associated with the Laplace and biharmonic equations [9], large-scale elliptic boundary value problems in three-dimensional axisymmetric domains [22], the diffraction of

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http://dx.doi.org/10.1016/j.amc.2016.08.021
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Rayleigh waves by fluid-saturated poroelastic alluvial valleys in a poroelastic half-space [28], the current-hole simulation in a cylindrical Tokamak [41], etc.

A significant number of boundary value problems in elasticity are characterized by incomplete boundary conditions, in the sense that a part of the boundary of the solution domain occupied by the elastic solid is under-specified since both the traction and the displacement vectors are unknown and have to be determined on this portion of the boundary. It is well-known that such inverse problems are in general ill-posed, in the sense that the existence, uniqueness and stability of their solutions are not always guaranteed [16]. The lack of complete boundary conditions, encountered in the case of inverse boundary value problems, is usually overcome by supplying additional information in the form of either internal displacement, strain or stress measurements, or over-specified boundary conditions on the aforementioned boundary. There are numerous important contributions in the literature, as well as various approaches, devoted to theoretical and numerical solutions of inverse boundary value problems in elasticity. For an extensive overview of inverse problems in elasticity over the last decades, we refer the reader to Bonnet and Constantinescu [6].

In general, two major classes of regularization methods are employed for the stable solution of inverse boundary value problems in linear elasticity, namely non-iterative/direct and iterative methods. Herein we refer to the first class mentioned above which is usually based on either the minimization of a Tikhonov functional (or, equivalently, the resolution of the normal equation) [48] or the decomposition of the matrix corresponding to the discretized system of equations, e.g. using the singular value decomposition (SVD) [18], which is successively used to solve a sequence of well-conditioned problems depending on the regularization parameter. Finally, the value of the regularization parameter and, consequently, the corresponding regularized solution, are selected using an appropriate criterion, such as the discrepancy principle (DP) [40], the generalized cross-validation (GCV) criterion [14] or the L-curve method (LC) [18].

Maniatty et al. [31] employed the finite element method (FEM) and a first-order spatial regularization scheme using the measurements of internal strains, as well as displacements, to solve for the boundary traction reconstruction in elasticity in terms of shape and magnitude. Schnur and Zabaras [47] presented the boundary condition reconstruction and the so-called keynode method, which consists of specifying a polynomial to represent the missing boundary condition. Spatial regularization and the BEM were also used by Zabaras et al. [53] to solve a similar problem. Later, Maniatty and Zabaras [30] applied the Bayesian statistical theory for general inverse problems to inverse elasticity problems and also compared it to the method proposed in [47]. Gao and Mura [12] formulated, for the first time, the Cauchy problem in two-dimensional elasticity in terms of both its integral representation and the Tikhonov regularization method (TRM). Martin et al. [38] combined the BEM and the SVD to determine the numerical solution of Cauchy problems in two-dimensional elasticity. However, in all of the above papers, no automatic criterion to efficiently select the regularization parameter was used. Both Turco [50] and Marin and Lesnic [33] used the BEM to discretize the problem along with the TRM completed by the GCV criterion and Hansen's L-curve rule, respectively, in order to make the solution process entirely automatic. The BEM-based system of linear equations was successfully solved via the conjugate gradient method (CGM) and a stopping criterion based on a Monte-Carlo simulation of the GCV by Turco [51]. The SVD, in conjunction with the BEM, was employed by Marin and Lesnic [34] and Marin et al. [37] to determine the numerical solutions to Cauchy problems in linear elasticity. Marin and Lesnic [35] and Marin [32] proposed the MFS in conjunction with the TRM for solving the Cauchy problem in two- and three-dimensional isotropic linear elasticity by inverting the associated normal equation, respectively. Bilotta and Turco [5] solved the Cauchy problem in two-dimensional isotropic linear elasticity by using a standard FEM approach, together with the TRM and the GCV criterion for the choice of the regularization parameter.

Encouraged by the studies performed by Wei et al. [52] and Marin et al. [36], who analysed the numerical solution of Cauchy problems associated with the two-dimensional Laplace and Helmholtz-type equations, and inverse boundary value problems in two-dimensional steady-state isotropic thermoelasticity, respectively, using SVD-based non-iterative regularization methods, in this paper we have decided to investigate thoroughly the application of these direct regularization methods to the stable solution of inverse boundary value problems in two- and three-dimensional linear elasticity, i.e. the TRM [48], the damped SVD (DSVD) and the truncated SVD (TSVD) [18]. Moreover, we analyse several criteria for the selection of the regularization parameter which can be chosen according to Morozov's DP [40], the GCV criterion [14] and Hansen's LC method [20]. Also, a comparison of the numerical results retrieved for every possible coupling regularization method-selection criterion is made.

The paper is organized as follows: In Section 2 we formulate mathematically the inverse problems under investigation. The MFS approach for inverse boundary value problems in two- and three-dimensional isotropic linear elasticity is presented in Section 3, whilst the SVD-based regularization methods mentioned above and the criteria for the selection of the regularization parameter are briefly presented in Section 4. The accuracy and stability of the numerical results obtained using these regularization methods and selection criteria are thoroughly analysed for four examples in Section 5. Finally, some conclusions are presented in Section 6.

2. Mathematical formulation

Consider a bounded domain $\Omega \subset \mathbb{R}^p$, where $p = 2, 3$ is the dimension of the space where the problem is posed, occupied by a homogeneous isotropic linear elastic material characterized by Poisson's ratio $\nu \in (0, 0.5)$ and the shear modulus $G > 0$. We also assume that the boundary $\partial\Omega$ of the solution domain Ω is either a smooth or a piecewise smooth curve $(p = 2)$ or surface $(p = 3)$.

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