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Robust finite-time stability and stabilization of uncertain Markovian jump systems with time-varying delay☆

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ABSTRACT

This paper deals with the robust finite-time stability and stabilization problems of uncertain stochastic delayed jump systems, where the uncertainty is in the form of additive perturbations and exists in the drift and diffusion sections simultaneously. Though perturbation, time-varying delay and Brownian motion existing at the same time, two conditions checking its robust finite-time stability are proposed by a mode-dependent parameter approach, which are different from some existing methods. Based on the proposed results, sufficient conditions for the existence of the state-feedback controller are provided with LMIs, which could be solved directly. It is seen that all the features of the underlying system such as time-varying delay, perturbation, diffusion, mode-dependent parameters and uncertain transition rate matrix play important roles in the system analysis and synthesis of finite-time stability. Finally, numerical examples are used to demonstrate the effectiveness and superiority of the proposed methods.

1. Introduction

As we know, many dynamical systems have their structures or parameters changed randomly, which are very suitable modeled to be Markovian jump systems (MJSs). Up to now, many important research topics of this kind of systems, such as stability analysis [1–4], stabilization [5–12], H_∞ control and filtering [13–18], output control [19–21], state estimation [22–25], adaptive control [26–28], synchronization [29,30], and so on, have emerged. Based on the various results proposed up to the present date, it is seen that a current trend is to study robust problems of delayed MJSs, whose motivation comes from the practical applications especially communication engineering. Very recently, reference [31] considered the robust stability and control problems of MJSs by exploiting a certain adjoint Lyapunov operator, where the uncertainty exists in terms of additive perturbations. There, time delay was not considered, and no environmental noise was added. It is very known that the presence of time delay often degrades system performance, even leads to instability, while environmental noise cannot be neglected in many dynamical systems. Moreover, it is seen from this reference that the proposed method is concerned about uncertain MJSs without time delay and environmental noise and not suitable for the problems presented

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for the systems to be considered in this paper. The main reasons are as follows. Firstly, the exploited Lyapunov functions between them are different. The Lyapunov functional to be used in this paper should consider the effects of time-varying delay, in which additional partitions are needed; Secondly, due to time-varying delay and environmental noise existing simultaneously, it will be seen that such additional partitions are important and complicate the system analysis and synthesis within solvable forms; Thirdly, but not the last, the additive perturbations included in diffusion section could also make the studied problems complicated, where some additional problems without in the above references should be considered carefully. Based on these facts, it is important and necessary to consider the related robust problems of uncertain delayed MJSs, where both delay and uncertainty exist in the drift and diffusion sections simultaneously.

On the other hand, by investigating most results on the stability in the literature, it is seen that they were mainly focused on the classical Lyapunov stability which guarantees the stability in an infinite-time interval. Different from the classical Lyapunov stability concept, finite-time stability is defined as the system state does not exceed a certain bound during a fixed finite-time interval. The introduction of such a stability concept is very necessary and important in many practical applications, where the related quantities need to fall into a range with specified bounds in a fixed-time interval. Up to present, many results, such as finite-time stability [32–36], stabilization [37–41], finite-time H_∞ control or filtering [42–44], finite-time consensus [45–47], input-output finite-time stability and stabilization [48,49], and so on, have emerged. Very recently, a new method depending on mode-dependent parameters [50] was proposed to deal with the finite-time stability of stochastic Markovian jump systems. Different from the traditional methods, the mode-dependent parameter approach has advantages in terms of having less conservatism. Moreover, the stabilization conditions were presented with coupled matrix inequalities and should be solved by using the N-mode algorithm. Though, the above proposed results are important and have superiorities, there are still more problems could be further considered. For example, when there is time delay, how to check its finite-time stability via a similar mode-dependent parameter method should be revisited. However, when a similar mode-dependent parameter approach is used, due to time-varying delay and uncertainty existing in the drift and diffusion sections simultaneously, some additional and new problems will emerge in the process of our results. The second problem but not the last one is how to make the obtained results with easily solvable forms, especially time delay and additive perturbation exist. By investigating the existing results about finite-time stability and similar topics as far as possible, it is found that there are few results about Markovian jump systems containing time delay and Brownian motion simultaneously. In recent reference [51], it is seen the given results are also coupled matrix inequalities. Moreover, the related time delay is constant. More importantly, some additional equation constraints coming from the term of Lyapunov function related to time delay should be satisfied, which cannot be solved easily. In other words, how to make the presented results with a easier and solvable forms should be considered carefully, especially time delay is contained. Based on the above all facts and demonstrations, due to these general conditions existing, some new and challenging problems will be brought and still remain to be solved. To our best knowledge, very few results are available to study the above problems. All the observations motivate the current research.

In this paper, we address the finite-time stability and stabilization for a class of uncertain stochastic delayed jump systems, where the uncertainty exists in the drift and diffusion parts in terms of additive perturbations simultaneously. It is said that the related problems to be considered are not just combination of the above each issues. The main contributions of this paper are summarized as follows: 1) Sufficient conditions checking the finite-time stability of the underlying system are presented with LMI forms, where a mode-dependent parameter approach is exploited successfully. Especially, some techniques in terms of additional inequalities and variables are proposed to deal with the above all problems resulting from such general issues; 2) Compared with the similar results about finite-time stability and stabilization of MJSs, the proposed methods have some advantages in terms of being solved easily. More importantly, the above equation constraints coming from the Lyapunov functional and being related to mode-dependent parameter approach have been removed; 3) Based on the established results, it is seen that time-varying delay, diffusion, perturbation and general TRMs are not just combined in this paper. The effects of such general issues are fully considered, which are proved to be very important in system analysis and synthesis.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. $\mathscr{E}[\cdot]$ means the mathematical expectation of $[\cdot]$. $\|\cdot\|$ refers to the Euclidean vector norm or spectral matrix norm. Ω is the sample space, $\mathcal{F}$ is the σ-algebras of subsets of the sample space and $\mathbb{P}$ is the probability measure on $\mathcal{F}$. In symmetric block matrices, we use "$*$" as an ellipsis for the terms induced by symmetry, diag $\{\cdots\}$ for a block-diagonal matrix, and $(M)^\star \triangleq M + M^T$.

2. Problem formulation

Consider a kind of uncertain stochastic Markovian jump systems with time-varying delay defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and described as

$$\begin{cases} dx(t) = (A(r_t)x(t) + A_h(r_t)x(t-h(t)) + E(r_t)w(t))dt \\ \qquad\qquad + (C(r_t)x(t) + C_h(r_t)x(t-h(t)) + F(r_t)w(t))d\omega(t) \\ y(t) = G(r_t)x(t) \\ x(t) = \phi(t), \forall t \in [-\bar{h}, 0] \end{cases} \tag{1}$$