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Stability of the permanent rotations of an asymmetric gyrostat in a uniform Newtonian field

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ABSTRACT

The stability of the permanent rotations of a heavy gyrostat is analyzed by means of the Energy-Casimir method. Sufficient and necessary conditions are established for some of the permanent rotations. The geometry of the gyrostat and the value of the gyrostatic moment are relevant in order to get stable permanent rotations. Moreover, the necessary conditions are also sufficient, for some configurations of the gyrostat.

1. Introduction

A gyrostat $\mathcal{G}$ is a mechanical system made of a rigid body $\mathcal{P}$ called the *platform* and other bodies $\mathcal{R}$ called the *rotors*, connected to the platform in such a way that the motion of the rotors does not modify the distribution of mass of the gyrostat $\mathcal{G}$. Due to this double spinning, the platform on the one hand and the rotors on the other, the gyrostat is also known with the name of *dual-spin* body.

In Astrodynamics, gyrostats play an essential role, since they are used for controlling the attitude dynamics of a spacecraft and for stabilizing their rotations. See, for instance, Cochran [8], Hall [15–17], Elipe and coworkers [10–13,20,26], Vera [37], Aslanov [4,5] and also Hughes [19] for further references.

Besides its practical interest, the rotational motion of a gyrostat is very interesting from a mathematical point of view. Indeed, principal moments of inertia and gyrostatic momenta may be considered as parameters in the Euler equations of motion and there is a wide variety of possible equilibria, trajectories and bifurcations even in the simplest case of a gyrostat in free motion, that is to say, under no external forces. The authors have been studying this case for several years, and one of the main results obtained is the proof that when the gyrostat motion is formulated in terms of the angular momentum components, this problem is equivalent to a parametric quadratic Hamiltonian [12], and for those class of quadratic Hamiltonians, the classification of equilibria and bifurcations in different regions of the parametric space are well studied [24,25,27].

A further step in the complexity of the problem, and in the approximation to a real one, is to consider the motion of a gyrostat under the attraction of a Newtonian field. For this problem, some authors have found approximated analytical solutions for particular cases [7,33] and other authors have studied the equilibria and their stability when the gyrostat is in circular orbit [34,35] or in the gravity field of a number of different rigid bodies [21,36]. In this paper we focus on the

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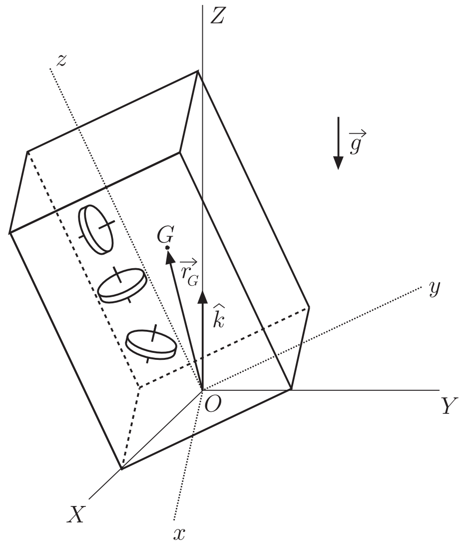

Fig. 1. Asymmetric gyrostat and reference frames.

stability of permanent rotations of a heavy gyrostat with a fixed point, that is to say when the gyrostat is under a uniform gravity field. For this case, both necessary and sufficient conditions of stability have been obtained by means of different methods. In this sense, Rumiantsev [32] and Anchev [1,2] gave sufficient conditions of stability for permanent rotations by constructing appropriate Lyapunov functions. In the particular case the center of mass lies on the first principal axis and the gyrostatic moment is directed along the same axis, Kovalev [22] derived sufficient conditions, that matched those of Rumiantsev, but also applied KAM theory to study the stability when the associated quadratic form of the perturbed Hamiltonian is not sign definite, but the necessary conditions are satisfied.

Previous results can also be derived and improved using the Energy-Casimir method [3,18,30,31] provided the system can be regarded as a Lie-Poisson one. Indeed, this method has been successfully used to study rigid body dynamics [6] and recently applied to study the stability of permanent rotations of a heavy gyrostat [14]. In this paper, the authors obtain, for a special class of permanent rotations, the same results previously derived by classical methods by Kovalev [23]. For the other permanent rotations, they provide sufficient stability conditions. However, these conditions are weak, as they do not depend on the gyrostatic moment. In this paper we obtain new sufficient conditions for all the permanent rotations in the case studied in [14] and also prove that, in some configurations of the moments of inertia, they are also necessary conditions. Besides, on a certain parametric plane, we determine regions for the existence of the equilibria, as well as bifurcation lines, since the stability depends on those parameters.

2. Equations of motion

Let us consider a gyrostat, consisting of a rigid asymmetric platform and three axisymmetric rotors. Each one of these rotors is aligned along one of the principal axis of the platform. The gyrostat is subject to a uniform and constant gravity field. We assume that the gyrostat has a fixed point O. Centered on this point, we consider two orthonormal reference frames (see Fig. 1):

- The inertial fixed reference frame $\mathcal{F}\{O, X, Y, Z\}$. The direction of the Z axis is opposite to the action line of the gravity field.
- The body frame $\mathcal{B}\{O, x, y, z\}$ fixed in the platform. The directions of these axes coincide with the principal axes of the gyrostat.

In the body reference frame $\mathcal{B}$, the tensor of inertia $\mathbb{I}$ of the gyrostat is diagonal, that is, $\mathbb{I} = \text{diag}(I_1, I_2, I_3)$. As we assume an asymmetric gyrostat, $I_1 \neq I_2 \neq I_3$. On the other hand, the total angular momentum of the gyrostat can be written as

$$\boldsymbol{H} = \boldsymbol{\pi} + \boldsymbol{l},$$

where $\boldsymbol{\pi}$ is the angular momentum of the whole gyrostat with rotors at relative rest (in other words, considering the gyrostat as a rigid body), and $\boldsymbol{l}$ is the gyrostatic momentum, that is, the relative angular momentum of the rotors with respect to the platform. Due to the gravity field, the gyrostat is under the action of a gravitational torque $\boldsymbol{N}$ about the fixed point O, given by

$$\boldsymbol{N} = \boldsymbol{r}_G \times m\boldsymbol{g} = -mg\, \boldsymbol{r}_G \times \hat{\boldsymbol{k}},$$