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Energy decay for a porous thermoelastic system with thermoelasticity of second sound and with a non-necessary positive definite energy

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ABSTRACT

In this paper we investigate a porous thermoelastic system where the heat conduction is given by Cattaneo's law and where the energy associated with the solution is not necessary positive definite ($\mu\xi = b^2$). We introduce a stability number χ and prove an exponential (resp. polynomial) decay result depending on χ.

1. Introduction

The analysis of the temporal behavior in porous thermoelasticity has attracted the attention of researchers shortly after Cowin and Nunziato [7] have introduced the linearized theory for elastic materials with voids. In the absence of body forces, the differential equations, in the general homogeneous and isotropic case, are

$$\begin{cases} \rho u_{tt} = \mu\Delta u + (\mu+\lambda)\nabla(\mathrm{div} u) + \beta\nabla\phi \\ \rho k\phi_{tt} = \alpha\Delta\phi - \tau\phi_t - \xi\phi - \beta \mathrm{div} u \end{cases} \tag{1.1}$$

where u is the displacement field, ϕ is the difference of the volume fraction and α, β, ρ, μ, λ, τ and ξ are constitutive coefficients.

To make system (1.1) dissipative, Cowin and Nunziato [7] and Cowin [8] assumed that the energy associated with system (1.1) is a positive definite quadratic form. Thus, the constitutive coefficients must satisfy the following inequalities

$$\mu \geq 0,\ \ \alpha \geq 0,\ \ \xi \geq 0,\ \ (2\mu + 3\lambda)\xi > 3\beta^2. \tag{1.2}$$

In the one-dimensional setting, system (1.1) becomes

$$\begin{cases} \rho u_{tt} = \mu u_{xx} + \beta\phi_x \\ J\phi_{tt} = \alpha\phi_{xx} - \beta u_x - \xi\phi - \tau\phi_t, \end{cases} \tag{1.3}$$

where we write μ instead of $2\mu + \lambda$. The assumption (1.2) then takes the form

$$\xi \geq 0,\ \ \alpha \geq 0,\ \ \mu\xi > \beta^2.$$

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Quintanilla [23] studied (1.3) and proved that the dissipation given by the viscous porosity is not powerful enough to produce an exponential stability, only slow decay has been obtained. Several dissipative mechanisms were considered in the succeeding contributions which intended to stabilize system (1.3) strongly. Quintanilla and coauthors [5,6,13,14,20,21,22] examine the coupling of system (1.3) with several dissipations. They combined temperature, elastic viscosity, porous viscosity and microtemperature and concluded that mixing temperature (resp. viscoelasticity) effect with microtemperature or with viscoporosity effects lead to a strong stability, otherwise the decay rate is slow. Soufyane [30] coupled system (1.3) with classical macrotemperature effect and replaced the damping $-\tau\phi_t$ by a viscoporous damping term of memory type $-\int_0^t g(t-s)\phi_{xx}(x,s)ds$ and proved an exponential (respectively polynomial) decay rate depending on the rate of decay of the relaxation function g. Messaoudi and Fareh [15,16] considered the system

$$\begin{cases} \rho_1 u_{tt} - k(u_x+\phi)_x + \theta_x = 0 & (0,L)\times(0,+\infty) \\ \rho_2\phi_{tt} - \alpha\phi_{xx} + k(u_x+\phi) - \theta + \int_0^t g(t-s)\phi_{xx}(x,s)ds = 0 & (0,L)\times(0,+\infty) \\ \rho_3\theta_t - \kappa\theta_{xx} + u_{xt} + \phi_t = 0 & (0,L)\times(0,+\infty) \end{cases}$$

where u, ϕ and θ are the transversal displacement, the volume fraction and the difference of temperature respectively. They improved the results obtained by Soufyane [30] and established a general decay result for a wide class of relaxation functions.

In all above mentioned problems, the heat equation dissipation is given through Fourier's law. This theory, as it is well known, predicts an infinite speed of heat propagation. To overcome this physical paradox, many theories have been developed. One of these theories, developed by Lord and Shulman [12], suggests that Fourier's law

$$q + \kappa\nabla\theta = 0,$$

be replaced by Cattaneo's law

$$\tau_0 q_t + q + \kappa\nabla\theta = 0,$$

where the positive constant τ_0 represents the time lag in the response of the heat flux to the temperature gradient and is referred to as the thermal relaxation time. According to this theory, heat propagation is to be viewed as a wave-like propagation rather than diffusion phenomenon. A wave-like thermal disturbance is referred to as second sound (where the first sound being the usual sound) and a nonclassical theory predicting the occurrence of such disturbances are known as thermoelasticity with finite wave speeds or second sound thermoelasticity.

In the one-dimensional case, Tarabek [31] treated a nonlinear system in both bounded and unbounded situations and established global existence results for small initial data. He also proved, without specifying the rate of decay, the strong convergence of derivatives of solutions to zero as t tends to infinity. Racke [25] looked into the system

$$\begin{cases} u_{tt} - \alpha u_{xx} + \beta\theta_x = 0 \\ \theta_t + \gamma q_t + \delta u_{tx} = 0 \\ \tau_0 q_t + q + \kappa\theta_x = 0, \end{cases} \tag{1.4}$$

with regular initial data and established an exponential decay results for boundary conditions modeling a rigidly clamped medium and with zero heat flux or constant temperature on the boundary. Racke [26] studied the multi-dimensional case of (1.4) together with Dirichlet-Dirichlet boundary conditions, in a bounded domain $\Omega\subset\mathbf{R}^n$, $(n=2,3)$ with smooth boundary $\partial\Omega$. He established an exponential decay result for solutions obeying the condition

$$\operatorname{rot} u = \operatorname{rot} q = 0.$$

For thermoelastic Timoshenko systems with second sound, we mention the work of Messaoudi et al. [19], in which they considered

$$\begin{cases} \rho_1\varphi_{tt} - (\sigma(\varphi_x+\psi))_x + \mu\varphi_t = 0 \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x+\psi) + \gamma\theta_x = 0 \\ \rho_3\theta_t + \kappa q_x + \gamma\psi_{xt} = 0 \\ \tau q_t + q + \kappa\theta_x = 0 \end{cases}$$

and established several exponential decay results, without restrictions on the coefficients, for both nonlinear as well as the linearized systems. Fernandez Sare and Racke [24] considered the system

$$\begin{cases} \rho_1\varphi_{tt} - k(\varphi_x+\psi)_x = 0 \\ \rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x+\psi) + \gamma\theta_x = 0 \\ \rho_3\theta_t + \kappa q_x + \gamma\psi_{xt} = 0 \\ \tau q_t + q + \kappa\theta_x = 0 \end{cases} \tag{1.5}$$

and proved that (1.5) is not exponentially stable even if the speed of propagation are equal $(\frac{k}{\rho_1} = \frac{b}{\rho_2})$ and even if an extra viscoelastic damping in the second equation is added. Santos et al. [28] considered (1.5) and introduced a new stability number

$$\chi = \left(\tau - \frac{\kappa\rho_1}{\rho_3}\right)\left(\rho_2 - \frac{b\rho_1}{\kappa}\right) - \frac{\tau\delta^2\rho_1}{\kappa\rho_3} \tag{1.6}$$