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Stability analysis and state feedback control of continuous-time T–S fuzzy systems via anew switched fuzzy Lyapunov function approach

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ABSTRACT

This paper deals with stability analysis and control design problems for continuous-time Takagi–Sugeno (T–S) fuzzy systems. First, a new membership-function-dependent switching law is proposed and a relaxation parameter is introduced into this switching law to guarantee a minimal dwell time between two consecutive switching. Compared to the existing methods, the most important point is that with the help of the dwell time, the discretized Lyapunov function (DLF) technique can be adopted. Then a new stability criterion of the T–S fuzzy system with less conservatism is derived based on a fuzzy discretized Lyapunov function (FDLF). Second, to estimate the domain of attraction (DA), an algorithm with less iteration steps is proposed. Based on the proposed switching method, sufficient conditions for existence of the state feedback controllers are presented via the switched non-parallel distributed compensation (non-PDC) scheme. The effectiveness of the proposed method is illustrated through three simulation examples.

1. Introduction

Over the last decades, Takagi–Sugeno (T–S) fuzzy models [1] have been widely exploited in nonlinear system modeling and control since it is an effective tool in approximating most complex nonlinear systems [2]. The main feature of the T–S modeling is that the nonlinear systems is described by a weighted sum of linear time invariant systems, where each linear system is a rule of fuzzy implication and the weights of the sum are nonlinear functions, called membership functions, which satisfy the sum-convex property [3]. Then, the nonlinear systems described by T–S fuzzy models can be analyzed using linear matrix inequalities (LMIs) formulations.

In the past decades, considerable efforts have been contributed to the T–S fuzzy systems. For T–S fuzzy control systems, many researchers have presented the conventional quadratic Lyapunov function approaches to find a constant positive definite matrix of a quadratic Lyapunov function satisfying the stability conditions of all subsystems [4–7]. However, it is obvious that the common quadratic approach leads to considerable conservativeness in that a common Lyapunov matrix should be found for all subsystems of the T–S fuzzy systems [8]. For nonlinear Markov jump systems, Cheng [9] considers a kind of Markov jump Lyapunov function. However, this Lyapunov function has the same disadvantage as common quadratic Lyapunov function. Therefore, for reducing the conservativeness, a kind of fuzzy Lyapunov functions which depend on the same

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membership functions as the original T–S fuzzy system have been developed. Nowadays, there has been a lot of interest in the use of the fuzzy Lyapunov functions for both continuous-time systems [8,10,11] and discrete-time systems [13–15]. Besides, some other fuzzy Lyapunov function approaches have been proposed for the stability analysis of the T–S fuzzy systems, such as piecewise quadratic Lyapunov functions [17], line-integral fuzzy Lyapunov function approach [12].

For the convenience in dealing with the nonlinear systems, numerous problems have been studied based on T–S fuzzy model. For example, state feedback control design problems are studied in [18,19], designing static output feedback controllers and dynamic output feedback controllers are given in [20,21], robust control design methods are presented in [23], adaptive fuzzy decentralized control problems are studied in [22,25,43] and non-fragile filtering and control problem have been investigated in [16,24]. For the control problem, a kind of non-parallel distributed compensation (non-PDC) control laws are widely applied in [26–28]. Besides, to reduce the conservatism, a kind of switched controllers is proposed for the fuzzy systems [29–31]. Especially, a new type of switched parallel distributed compensation (PDC) controllers which are switched based on the values of membership functions are proposed in [29]. However, for the fact that the switching law in [29] is based on the membership functions and has no minimal dwell time, the famous Metzler matrix method [34] and average dwell time method [35,36] cannot be applied to the fuzzy systems with switching control. This is the main motivation of the present study.

On the other hand, how to estimate the domain of attraction (DA) is another important issue in stability analysis of nonlinear systems. For a Lyapunov function $V(x(t))$ which guarantees the local stability of the equilibrium, any sublevel set of the Lyapunov function is an inner estimate of the DA if the set belongs to the region where $V(x(t)) > 0$ and $\dot{V}(x(t)) < 0$ hold for all $x(t) \neq 0$[37]. Based on the Lyapunov theory, some preliminary results in obtaining such estimates are proposed in [37]. Nowadays, based on the fuzzy Lyapunov function method, some systematic approaches to estimate the DA for continuous-time T–S fuzzy systems have been investigated in [8,10,38,41,42].

Motivated by the aforementioned, in this paper, the problem of stability analysis and stabilization for a class of T–S fuzzy systems are investigated. First, for T–S fuzzy systems, a kind of membership-function-dependent switching law is proposed. Compared to the existing switched or piecewise controllers method in [29–31], it is proved that the proposed switching signal has a minimal dwell time. It is worth noting that the existence of minimal dwell time is one of the most important points in this paper. With the help of the minimal dwell time, discretized Lyapunov function (DLF) technique can be adopted for less conservatism. Second, a more effective algorithm to estimate the DA for continuous-time T–S fuzzy systems is obtained based on the new stability condition. Third, referring to the non-PDC control scheme in [27], the state feedback control problem for continuous-time T–S fuzzy systems is solved via a new switched non-PDC controller. Finally, three simulation examples are illustrated to show the effectiveness of the proposed method.

The paper unfolds as follows: in Section 2, the T–S fuzzy systems and some definitions are given. The stability analysis and state feedback control of continuous-time T–S fuzzy systems are considered in Section 3. The stability analysis and an algorithm estimating the DA are proposed in Section 3.1 and the switched non-PDC state feedback controller is obtained in Section 3.2. In Section 4, three simulation examples are given to illustrate the effectiveness of the new proposed method. Finally, conclusions are presented in Section 5.

Notation. For a matrix P, P^T denotes its transpose and $He(P) \triangleq P + P^T$. $P = [p_{ij}]_{m\times n}$ denotes $P \in \mathbb{R}^{m\times n}$ and p_{ij} is the element on row i column j. $P > 0$ and $P < 0$ denote positive definite and negative definite, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. The notation $\|x\|_2$ refers to the Euclidean vector norm of vector $x \in \mathbb{R}^n$. Denote

$$P_{h(t)} = \textstyle\sum_{i=1}^{r} h_i(t)P_i, \quad P_{h(t)h(t)} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(t)h_j(t)P_{ij} \tag{1}$$

2. Preliminaries and problem statement

Consider a class of continuous-time T–S fuzzy control system which can be described as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \tag{2}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state, $u(t) \in \mathbb{R}^{n_u}$ is the control input. $\mathbb{I} = \{1, 2, \dots, r\}$, r is the number of IF-THEN rules. A_i, B_i are constant matrices of the appropriate dimensions. $z(t) = [z_1(t), z_2(t), \dots, z_q(t)]^T$ is the vector containing premise variables in the fuzzy inference rule. Besides, $h_i(t)$ are the normalized membership functions for ith rule fulfilling the following properties

$$h_i(t) \geq 0, \quad \sum_{i=1}^{r} h_i(z(t)) = 1, \quad \sum_{i=1}^{r} \dot{h}_i(z(t)) = 0, \quad i \in \mathbb{I} \tag{3}$$

where $h_i(z(t))$ are said to be the normalized membership functions. Considering the time derivative of the normalized membership functions, we assume that

$$|\dot{h}_i(z(t))| \leq \phi_i, \quad i \in \mathbb{I} \tag{4}$$

For the presentation convenience, denote $h(z(t)) = [h_1(z(t)), h_2(z(t)), \dots, h_r(z(t))]^T$, besides $h(z(t))$ and $h_i(z(t))$ are denoted as $h(t)$ and $h_i(t)$, respectively.