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Compressible dusty gas along a vertical wavy surface

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ABSTRACT

This analysis deals with the numerical solutions for the compressible natural convection flow of two-phase dusty gas. In particular, it gives the solutions for the flow having spherical particles suspended in the gas (air) over the surface with sharp lateral curvatures. The governing equations are converted into dimensionless equations by using a set of suitable continuous transformations and solved through the implicit finite difference method. The effect of compressibility, dusty gas and sinusoidal waveform are discussed in detail in terms of local heat transfer rate, skin friction coefficient, velocity and temperature distributions. This investigation reveals the fact that the air–particle mixture reduces the rate of heat transfer, significantly.

1. Introduction

Compressible fluid flows are of great importance in mechanical, aerospace and chemical engineering problems and the understanding of the influence of compressibility on the flow formation comes out to be a much needed information in this context. The most obvious applications of compressible fluid flow theory are found in the design of aircrafts and also in the design and operation of devices used in gas turbines, reciprocating engines, stream turbines, natural gas transmission lines and combustion chambers [1]. For high Reynolds number, the problem of compressible laminar boundary layer flow along a flat surface has been studied by Kuerti [2] and Young [3]. Moore [4] developed the first-order deviations of the velocity and temperature distributions for the compressible, quasi-steady state, boundary layer flow over a semi-infinite flat plate. The theory of compressible boundary layer flow has been much refined and generalized since then. Extensive reviews of the literature on this subject have been given by Stewartson [5], Gross and Dewey [6], Curle [7], Herwig [8], Kumari and Nath [9–11],van Oudheusden [12], Gersten and Herwig [13], Schlichting and Gersten [14], Hossain et al. [15] and Hossain and Pop [16].

Boundary-layer flows of gas–particle mixtures have attracted numerous experimentalists due to their wide range of applications in various problems of atmospheric, engineering and physiological fields (see [17]). In this regard, the problem related to the boundary layer flow of dilute gas–particle suspension have been investigated by many researchers, for example, [18–25]. Out of these, Singleton [20] was the first to extend the Marble's analysis [19] to compressible case where the density for both phases (carrier phase and particle phase) may vary. In this paper, the author developed the governing boundary-layer equations under the assumption of particular form of viscosity-temperature relation (i.e $\mu^*/\mu^*_\infty = (T^*/T^*_\infty)^{0.5}$) and solved the two-phase model for Stokes's relation. Based on the analysis [20], Wang and Glass

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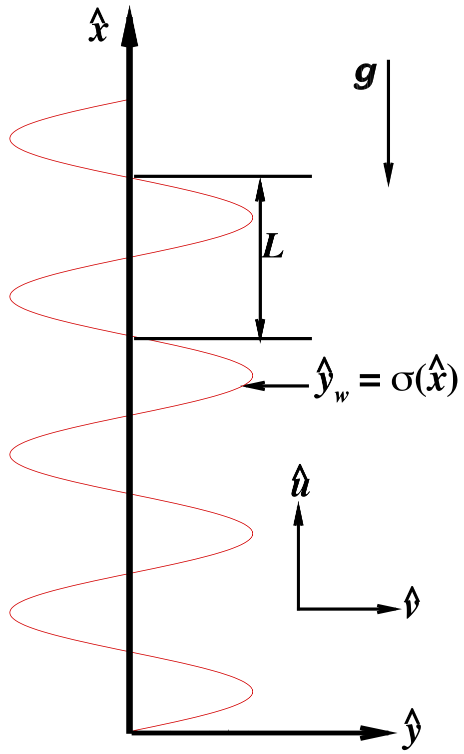

Fig. 1. Schematic of the problem.

[22] presented the more generalized form of governing equations by including the more reasonable expressions for the interaction of two-phase and the gas viscosity. In [22], the authors solved the problem of compressible dusty fluid for Stokian as well as for non-Stokian case for wide range of viscosity-temperature relation (i.e $\mu^*/\mu^*_\infty = (T^*/T^*_\infty)^\omega$; $0.5 \leq \omega \leq 1.0$, where ω being the power index).

In the present analysis, we presented the numerical solutions for compressible dusty gas along a vertical wavy surface for generalized form of temperature-viscosity relation. Owing to the complexity of the geometry of the problem and the fully compressible boundary layer equations for two-phase dusty fluid flow, a mathematical model is developed, which is possible to justify to a certain extent of physical grounds. Certain assumptions have been made to confine the influence of compressibility of carrier fluid into the boundary layer region; provided the main stream remains incompressible (i.e $\omega = \delta = 0.0$). The governing boundary equations are reduced to a convenient form by the introduction of primitive variable formulation and the resulting parabolic partial differential equations are solved by finite difference method together with Gaussian elimination technique. Numerical results of the two-phase problem are displayed in the form of wall shear stress, heat transfer rate, velocity and temperature profiles by varying several controlling parameters.

2. Analysis

Consider a vertical plate with transverse sinusoidal undulations situated in two-phase dusty fluid, as illustrated in Fig. 1. In particular, we assume that the surface profile is given by (for details see [26]):

$$\hat{y}_w = \sigma\left(\hat{x}\right) = \hat{a}\sin\left(\frac{2\pi\hat{x}}{L}\right) \tag{1}$$

where $\hat{a}$ is the amplitude of the transverse surface wave and L the characteristic length associated with the wave (over hats denote the dimensional quantities). Consideration has been given to steady, compressible dusty fluid flow and the Boussinesq approximation is employed. Both the gas and the particle cloud along the vertical wavy surface are supposed to be static at the beginning and the number density of the particles is taken as uniform throughout the flow. Another assumption is that the dust particles are spherical in shape and uniform in size so that the conservation equations given by Saffman [18] remains valid. It is assumed that the surface temperature of the vertical wavy surface, T_w, is uniform and $T_w \gg T_\infty$, where T_∞ is the ambient fluid temperature. The governing equations of the convective flow along the vertical surface can be written in dimensional form as (for details see [18,23,27]):