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Statistics for anti-synchronization of intracellular calcium dynamics☆

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ABSTRACT

The anti-synchronization of calcium oscillation between cytosol and calcium store in intracellular calcium oscillation system with non-Gaussian noises and time delay are studied by means of second-order stochastic Runge–Kutta type algorithm. Basic on statistic, the normalized global synchronization error of Ca^{2+} concentration in cytosol and calcium store is simulated, the results exhibit, it shifts into single trough from multi troughs structure as parameter p (which is used to control the degree of the departure from the non-Gaussian noise and Gaussian noise) or density of non-Gaussian noises increases, however, it shifts into multi troughs from single trough structure as correlation time of non-Gaussian noises increases. In addition, the short, moderate, and long time delay respectively induce anti-synchronously quasi-periodic, synchronously quasi-periodic, and chaotic oscillation.

1. Introduction

Ca^{2+} is an ubiquitous and versatile second messenger that transmits information through changes of the cytosolic Ca^{2+} concentration, namely, Ca^{2+} signaling pathway translates external signals into intracellular responses by increasing the cytosolic Ca^{2+} concentration in a stimulus dependent pattern. Specifically, the increasing of concentration can be caused either by Ca^{2+} entry from the extracellular medium through plasma membrane channels, or by Ca^{2+} release from the internal calcium store.

There are a variety of channels showing calcium-induced calcium release and a variety of models [1–4] so far. In many studies on intracellular calcium oscillation(ICO) system, some phenomena have been found such as stochastic resonance [5,6], reverse resonance [6–8], oscillatory coherence [9], coherence resonance [7], resonant activation [10], bistability solutions with hysteresis [11,12], calcium puffs [13], various spontaneous Ca^{2+} patterns [14], stochastic backfiring [15], dispersion gap and localized spiral waves [16], stability transition [17], calcium wave instability [18], colored noise optimized calcium wave [19], periodic square calcium wave [20], etc.

Importantly, ICO has been intensively studied by Perc group [21–24] and Falcke group [15,16,25–29]. Among, Perc group have found that noise and other stochastic effects indeed play a central role [21,22], in the transmission processes of intra-

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cellular Ca^{2+}, there may be non-Gaussian noise [29]. Recently, we have studied ICO system driven by non-Gaussian noises [7,8], then found that ICO exhibits synchronous or anti-synchronous oscillation between cytosol and calcium store [7]. Interestingly, this kind of synchronization and anti-synchronization oscillation can regulate the calcium concentration in cytosol and calcium store [30], then can change the calcium wave across cells, so that can adjust finally the signal transmission across cells. Thus, in this paper, we compute the normalized global synchronization error of Ca^{2+} concentration in cytosol and calcium store to study anti-synchronous ICO.

First, according to Refs. [7,8], the ICO model with non-Gaussian noises and time delay is presented. Then, the normalized global synchronization error is computed. Finally, conclusions are obtained.

2. The model for ICO with non-Gaussian noises and time delay

Taking into account same time delay τ in processes of active and passive transport of Ca^{2+} in a real cell, the Langevin equations of ICO system read as follows [7,8]:

$$d_t x = A_1(x; x_\tau, y_\tau) + B_1(x; x_\tau, y_\tau)\eta_1(t), \tag{1}$$

$$d_t y = A_2(x, y; x_\tau) + B_2(x, y; x_\tau)\eta_2(t), \tag{2}$$

with

$$A_1(x; x_\tau, y_\tau) = v_0 + v_1\beta_0 - v_2 + v_{3\tau} + k_f y_\tau - kx, \tag{3}$$

$$A_2(x, y; x_\tau) = v_{2\tau} - v_3 - k_f y, \tag{4}$$

$$B_1(x; x_\tau, y_\tau) = \sqrt{v_1^2\beta_0^2 + 2v_1\beta_0\lambda W + W^2}, \tag{5}$$

$$B_2(x, y; x_\tau) = \sqrt{\frac{v_{2\tau} + v_3 + k_f y}{V}}, \tag{6}$$

$$W(x; x_\tau, y_\tau) = \sqrt{\frac{v_0 + v_1\beta_0 + v_2 + v_{3\tau} + k_f y_\tau + kx}{V}}, \tag{7}$$

and

$$v_2 = \frac{V_2 x^2}{x^2 + k_1^2}, v_3 = \frac{V_3 x^4 y^2}{(x^4 + k_2^4)(y^2 + k_3^2)}, \tag{8}$$

$$v_{2\tau} = \frac{V_2 x_\tau^2}{x_\tau^2 + k_1^2}, v_{3\tau} = \frac{V_3 x_\tau^4 y_\tau^2}{(x_\tau^4 + k_2^4)(y_\tau^2 + k_3^2)}. \tag{9}$$

Here, x and y denote concentration of free Ca^{2+} of cytosol and calcium store in a cell, respectively. The rate v_2 and v_3 refer, respectively, to pumping of Ca^{2+} into the calcium store and to release of Ca^{2+} from store into cytosol in a process activated by cytosolic Ca^{2+}. $v_{2\tau}$ is v_2 with time delay, and $v_{3\tau}$ is v_3 with time delay. $W = W(x; x_\tau, y_\tau)$, $x_\tau = x(t-\tau)$, $y_\tau = y(t-\tau)$. λ denotes cross-correlation degree of internal and external noise before merger [12].

The noises $\eta_1(t)$ and $\eta_2(t)$ in Eqs. (1) and (2) are considered as non-Gaussian noises [8] which are characterized by the following Langevin equation [31]:

$$\frac{d\eta_i(t)}{dt} = -\frac{1}{\tau_1}\frac{d}{d\eta_i}V_{ip}(\eta_i) + \frac{\sqrt{2D}}{\tau_1}\xi_i(t), i = 1, 2. \tag{10}$$

Where $\xi_i(t)$ is a standard Gaussian white noise of zero mean and correlation $\xi_i(t)\xi_i(t') = \delta(t-t')$.$V_{ip}(\eta_i)$ is given by

$$V_{ip}(\eta_i) = \frac{D}{\tau_1(p-1)}\ln[1 + \frac{\tau_1}{D}(p-1)\frac{\eta_i^2}{2}], \tag{11}$$

and the statistical properties of non-Gaussian noise $\eta_i(t)$ is defined as

$$\langle\eta_i(t)\rangle = 0, \tag{12}$$

$$\langle\eta_i^2(t)\rangle = \begin{cases} \dfrac{2D}{\tau_1(5-3p)}, p \in (-\infty, \frac{5}{3}), \\ \infty, p \in [\frac{5}{3}, 3), \end{cases} \tag{13}$$

where τ_1 denotes the correlation time of the non-Gaussian noises $\eta_i(t)$, and D denotes the noise intensity of Gaussian white noise $\xi_i(t)$. The parameter p is used to control the degree of the departure from the non-Gaussian noise to Gaussian noise. The distribution of the noise is Gaussian for $p = 1$, non-Gaussian with long tail for $p > 1$, and characterized by a "more than Gaussian" cutoff for $p < 1$. Here, in order to study easily, supposing noises $\xi_1(t)$ and $\xi_2(t)$ have same strength D, and non-Gaussian noises $\eta_1(t)$ and $\eta_2(t)$ have same p and correlation time τ_1.