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Sliding mode control of Markovian jump systems with incomplete information on time-varying delays and transition rates

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ABSTRACT

This paper is concerned with robust sliding mode control (SMC) for continuous-time Markovian jump delay systems with incomplete transition rates (TRs), the time-varying delays are unclear and just bounded sometimes. Two methods are employed to investigate this control issue. One is movement decomposition method by which reduced-order sliding mode dynamics is obtained; the other is via sliding mode observer and full-order sliding mode dynamics is obtained on the estimation space. In both cases, sufficient conditions are established in terms of a set of coupled linear matrix inequalities (LMIs) to ensure the sliding mode dynamics to be mean-square exponentially stable; moreover, novel sliding mode controllers, which need not full knowledge on time-varying delays, are synthesized to guarantee the reachability of the prescribed sliding surface. Finally, numerical examples are provided to illustrate the effectiveness of the proposed methods.

1. Introduction

Since introduced by Krasovskii and Lidskii [1], Markovian jump systems (MJSs) have attracted considerable attention. As a special class of hybrid systems, it is very appropriate to model physical systems with abrupt structural changes due to, for example, sudden environment disturbance, unexpected changes of operating points in nonlinear systems, ect. In the past few decades, many results on MJSs have been reported; see, e.g. [2–8]. Recently, an interest in the uncertainty of TRs in the jumping process evoked due to the possibility of getting full information on TRs and high expense, many research activities have been attracted, such as in [9], sufficient conditions for stochastic stability and stabilization of Markovian jump linear systems with partly unknown TRs via LMIs formulation is developed; in [10], the authors studied the stabilization of mode-dependent singular MJSs with generally uncertain TRs, etc., see, e.g. [11–15]. While the field is not fully investigated yet, so it is significant to further study MJSs with incomplete TRs.

Appeared in the 1950s [16], SMC has been regarded as an important robust control strategy for various control systems [17,18], also as an important nonlinear robust control strategy and its nonlinear performance lies in control of discontinuity. The difference between this kind of control strategy from others is that the structure of the system is not fixed, but in the dynamic process, it changes in accordance with the state of the system, forcing the system trajectories moving according to

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the preliminary designed sliding mode. There are several attractive characteristics of SMC, such as fast response, superior transient performance and in particular, the rejection of disturbances and the insensitivity with respect to the variations of the plant parameters. Such as in [19], fuzzy logic systems-based integral sliding mode fault-tolerant control for a class of uncertain non-linear systems is investigated; and SMC design for linear systems subject to quantization parameter mismatch is studied in [20]; including robust H_∞ control for neutral-type systems via sliding mode observer [21]. On the other hand, SMC of MJSs is also an active field, like the author considered robust stabilization of Markov jump linear time-delay systems with generally incomplete TRs in [11]. However, the state information may not always be available in practice. Thus, observer-based feedback control problem for dynamical systems attracted tremendous attention, so does it for observer-based SMC of MJSs. Recently, the authors in [22] studied fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach; and they further investigated observer-based adaptive SMC for nonlinear MJSs in [23]; also, a sliding mode approach to H_∞ non-fragile observer-based control design for uncertain Markovian neutral-type stochastic systems is considered in [24].

Time-delay, which is encountered in various physical process, chemical process, and it is considered as the major cause of instability and poor performance of dynamical systems [25]. Much attention has been given to the investigation of time-delay systems. Also, notice that in [26], the SMC for a class of MJSs with mixed mode-dependent time-varying delays is considered and less conservative delay-dependent condition is derived, but in this paper full knowledge on the time-varying delays and the complete information on TRs are assumed, so what if those information are not exactly known.

In this paper, the SMC problem is studied for a class of continuous-time MJSs with mode-dependent time-varying delays. The issues involved here are delays uncertain and TRs incomplete. We will propose a novel method to design sliding mode controllers and give less conservative stability results. The content of this paper is organized as follows: problem statements and preliminaries are presented in Section 2. In Section 3, LMI conditions are derived to guarantee the existence of the linear sliding surface and mean-square exponential stability of the reduced-order sliding mode dynamics, then a sliding mode controller is proposed to guarantee the sliding motion. In Section 4, an observer-based integral-type sliding surface is designed, and LMI conditions are derived to guarantee the mean-square exponential stability of sliding mode dynamics and feasible controller is designed to ensure the system trajectories satisfy the reaching condition. Numerical examples are given to illustrate the effectiveness of the results in Section 5 and the conclusions are drawn in Section 6.

Notions: Throughout this paper, matrices, if not explicitly stated, are supposed to have compatible dimensions. The notion $X > 0$ ($X \geq 0$) means that X is symmetric positive definite (semi-positive definite) matrix. I and 0 are used to represent an identity matrix and zero matrix of appropriate dimensions. $\|\cdot\|$ refer to the Euclidean vector norm or spectral matrix norm. Let $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ be the minimum eigenvalue and maximum eigenvalue of a matrix respectively. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the σ-algebra of Ω and $\mathcal{P}$ be the probability measure on $\mathcal{F}$. $\mathbf{E}(\cdot)$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. The symmetric elements of the symmetric matrix are denoted by $*$. $sym\{P\}$ represents $P + P^T$.

2. Problem formulation and preliminaries

In this paper, we consider the following MJS fixed on the probability space:

$$\begin{cases} \dot{x}(t) = (A(r_t) + \Delta A(r_t, t))x(t) + (A_d(r_t) + \Delta A_d(r_t, t))x(t - d(r_t, t)) + B(r_t)(u(t) + f(x(t), t, r_t)), \\ y(t) = C(r_t)x(t), \\ x(t) = \phi(t) \qquad t \in [-d, 0]. \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^q$ is the system output, $f(x(t), t, r_t))$ is the system nonlinear perturbation, $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $C(r_t)$ are known real matrices of the random process $\{r_t, t \geq 0\}$; $\Delta A(r_t, t)$ and $\Delta A_d(r_t, t)$ are the system parameter uncertainties. The jumping process $\{r_t, t \geq 0\}$ is a homogenous Markov chain with right continuous trajectories, taking values in a finite state-space $\mathcal{S} = \{1, 2, \ldots, s\}$ with the following mode transition probabilities:

$$\mathbf{Pr}\{r_{t+\triangle} = j | r_t = i\} = \begin{cases} \pi_{ij} \triangle + o(\triangle), & if \quad i \neq j, \\ 1 + \pi_{ii} \triangle + o(\triangle), & if \quad i = j, \end{cases} \tag{2}$$

where $\triangle > 0$ and $\lim_{\triangle \to 0} o(\triangle)/\triangle = 0$, $\pi_{ij} > 0 (i \neq j)$, is the TR from mode i at time t to mode j at time $t+\triangle$, and for each $i \in \mathcal{S}$, there is $\pi_{ii} = -\sum_{j \neq i} \pi_{ij} < 0$.

In this paper, the TRs are considered to be partly known, i.e., some π_{ij} in the TR matrix $\mathbf{\Pi} \triangleq [\pi_{ij}]_{s \times s}$ are inaccessible for real systems. For instance, the TR matrix of the system (1) with s modes may be expressed as:

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & ? & ? & \cdots & \pi_{1s} \\ ? & \pi_{22} & \pi_{23} & \cdots & ? \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ ? & \pi_{s2} & ? & \cdots & \pi_{ss} \end{bmatrix},$$

whrer "?" means the TR π_{ij} is completely unknown.