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Axisymmetric solutions for the multi-layered transversely isotropic piezoelectric medium

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A B S T R A C T

By introducing the precise integration algorithm (PIA) and the technique of the mixed variable formalism accounting for any number of layers, variations of components of the axisymmetric piezoelectric field including elastic displacement, vertical normal stress, electric potential and electric displacement along the axis of symmetry in the cylindrical coordinate system are explored. The piezoelectric field is induced by the vertical loadings of mechanical or electric types uniformly distributed over a circular region. With the aid of the mixed variable formulations, along with the Hankel integral transform and the corresponding matrix algebraic operations, the governing partial differential equations of equilibrium expressed in terms of displacements and electric potential are reduced to the first order ordinary differential matrix equations. As a highly accurate algorithm, the PIA is provided to evaluate the obtained ordinary differential matrix equations in the transformed domain. Both mechanical and electrical quantities in the physical domain can be acquired by taking the inversion of the Hankel integral transform. An example as benchmark is illustrated to examine the applicability and performance of the proposed technique compared with results in the literature. Numerical examples are used to demonstrate the influence of the degree of the material anisotropy and stratified parameters.

1. Introduction

In recent years, with the rapid development of modern technology and material science innovative and smart structures and systems are attracting more and more attention because of their high performance in technological applications. With regards to their inherent feature of transforming energy between electric and mechanical types, piezoelectric materials have received a considerable amount of research. Owing to the characteristic phenomena, piezoelectric materials are now widely used as electromechanical devices, sensors and actuators in intelligent structures. Generally speaking, the piezoeffect materials tend to be anisotropic with respect to the piezoelectric properties. In particular, the present paper is concerned with one of the simplest types of anisotropy-transverse isotropy, which is perhaps the most technologically important material in piezoelectricity. To motivate the investigations of these stratified cross-anisotropic piezoelectric materials and predict

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reliable conclusions, it is necessary to analyze theoretically and study more profoundly the behavior of the layered piezoelectric media subjected to external forces of mechanical or electric types from a mechanical–electrical coupling view point.

Research on general solutions of multi-layered piezoelectric material structures has been attracting much attention and many important achievements have been made on the subject. Cady [1] pointed out that the piezoelectric effect of certain crystalline materials was able to generate an electrical charge in proportion of an externally applied force and to induce an expansion on the piezoelectric material in proportion to an electric field parallel to the direction of polarization. Based on the so-called state space formulation, Sosa and Castro [2] investigated the response of structures formed by assembling layers of various materials possessing very different elastic and electric characteristics under arbitrary loading conditions. Chen [3] as well as Chen and Lin [4,5] formulated Green's functions of infinite body for 3D problems in anisotropic piezoelectric materials and gained their derivatives of the first and second degree as the contour integrals over the unit circle by using the three-dimensional Fourier transforms. But the involved computation was cumbersome. Dunn [6] created a solution to Green's functions for an infinite transversely isotropic piezoelectric solid by virtue of taking Radon transforms, coordinate transformation and evaluation of residues in sequence. But the expressions of this solution were very complicated. It was difficult to be verified and inconvenient to be used. By employing potential functions, Wang and Chen [7] gave an explicit solution for the space axisymmetric problem in transversely isotropic piezoelectric material. Wang and Zheng [8] derived the general solution to governing equations of three-dimensional problems in a transversely isotropic piezoelectric medium. Liang et al. [9] exploited the Stroh formalism to acquire an exact general solution for an infinite, anisotropic piezoelectric medium with an elliptic inclusion. Based on Green's functions and their first partial derivatives, Chen and Lin [10] presented boundary integral formulae in the direct boundary element method for the analysis of three-dimensional piezoelectric boundary value problems. With the help of potential functions, the explicit closed-form expressions of the infinite-body and half-space Green's functions for a transversely isotropic piezoelectric medium were illustrated by Dunn and Wienecke [11,12]. Lee and Saravanos [13,14] proposed generalized discrete layer approaches for smart thermopiezoelectric beam structures and presented the development of comprehensive mechanics for the analysis of piezoelectric composite structures. Michelitsch [15] independently derived Green's function for the transversely isotropic piezoelectric space utilizing the method of integral transforms. According to the method of the image source, Ding et al. [16] conducted a systematical study on the problem of point force and point charge applied in the interior of an infinite two-phase transversely isotropic piezoelectric solid and gained overall solutions. By virtue of the potential theory method, Chen et al. [17] suggested a closed-form solution for the problem of a rigid conical and electrically conductive punch indenting a transversely isotropic piezoelectric half-space. Karapetian et al. [18] made use of the potential theory to acquire Green's functions of infinite three-dimensional transversely isotropic piezoelectric materials in a more direct and simpler way. With the aid of the Stroh formalism and two-dimensional Fourier transform in combination with Mindlin's superposition method, Pan and Yuan [19] studied the three-dimensional Green's function in the anisotropic piezoelectric half space and bimaterials. Pan and Tonon [20] provided explicit solutions for the extended Green's displacements in a 3D general anisotropic piezoelectric solid. Considering the satisfaction of the Maxwell static electricity equation, Wang et al. [21] proposed a model for the free vibration analysis of a piezoelectric coupled circular plate. Wang and Noda [22] took advantage of integral transform techniques to explore the fundamental solutions for multi-layered piezoelectric materials and structures under the action of axisymmetric strain state of deformation. The monographs by Ikeda [23] and Ding and Chen [24] contained extensive technical literatures regarding the macroscopic piezoelectric materials. Pan [25,26] introduced the generalized Mindlin's solution for an anisotropic and piezoelectric half-space with general boundary conditions on the surface and calculated the quantum dot induced by elastic and piezoelectric fields in certain semiconductor substrates. Wang et al. [27] applied the state vector method to solve the axisymmetric transversely isotropic piezoelectric problems in a system of cylindrical coordinates. Gautschi [28] found that piezoelectric materials had been widely used as sensors and actuators considering their advantages of smaller size and weight and higher bandwidth capabilities as compared with other devices. Liu and Fan [29] proposed a piezoelectric boundary element method to analyze 2-D thin piezoelectric structures with very small thickness-to-length ratios and solved the nearly singular integrals by transforming integrals into summations of several polynomial fractions. According to Pan [30], six different interface models were demonstrated analytically for the 2D and 3D anisotropic and piezoelectric bimaterials Green's function produced by point force or point charge. Taciroglu et al. [31] obtained the general solutions of a laminated circular piezoelectric cylinder subjected to axisymmetric mechanical and electrical loads by a semi-analytical finite element method. Yang et al. [32] revealed a general method of solving the problem of the 3D fundamental response in a multilayered anisotropic piezoelectric structure subjected to a point force and point charge. Ding et al. [33] summarized Green's function solutions and boundary element method for both 2D and 3D transversely isotropic piezoelectric materials. Xu et al. [34] utilized a generalized method brought forward by Gregory and Wan to solve transversely isotropic piezoelectric circular plates in axisymmetric deformations due to prescribed stress and electric displacement edge data. Li and Wang [35] introduced a simple and convenient method to obtain Green's functions of the infinite anisotropic piezoelectric media. The books by Qin [36] and Yang [37] summarized the boundary element analysis of fracture of electro-elastic and electro-magneto-elastic solids. Yang [38] carried out a study on the axisymmetric indentation of a semi-infinite, transversely isotropic piezoelectric material by a rigid, conducting indenter of arbitrary-axisymmetric profile. Xu et al. [39] utilized a systematic method to explore general solutions for three-dimensional transversely isotropic piezoelectricity. By expressing the displacements and electric potential as suitable polynomials in the radial coordinate with the coefficients being functions of the axial coordinate, Li et al. [40] gave an explicit solution for the problem of functionally graded piezoelectric materials plates under bending and tension. The derivation of the fundamental