Applied Mathematics and Computation 290 (2016) 507–520

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

The stability of stochastic coupled systems with time delays and time-varying coupling structure

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A R T I C L E I N F O

Keywords:
Time-varying coupling structure
Exponential stability
Coupled oscillators
Stochastic coupled systems

A B S T R A C T

This paper is concerned with the pth moment exponential stability of stochastic coupled systems with time delays and time-varying coupling structure (SCTVCS). The coupling structure is time-varying first. Second, the method of graph theory has been generalized from a system established on a digraph with constant coupling structure to a digraph with time-varying coupling structure successfully. Combining graph theory with Lyapunov method, a systematic method is given to construct a Lyapunov function for SCTVCS. Several pth moment exponential stability criteria are presented, including a Lyapunov-type theorem, and some sufficient criteria in the form of coefficients. Furthermore, the theoretical conclusions are applied to the stochastic coupled oscillators with time-varying coupling structure, and two stability criteria are obtained. Finally, we give two numerical examples to illustrate the effectiveness of our results.

1. Introduction

In this paper, we consider stochastic coupled systems with time delays and time-varying coupling structure (SCTVCS) as follows:

$$\mathrm{d}x_k(t) = \left[f_k(x_k(t), x_k(t-\tau), t) + \sum_{h=1}^{l} a_{kh}(t) H_{kh}(x_k(t), x_h(t)) \right] \mathrm{d}t + g_k(x_k(t), x_k(t-\tau), t)\mathrm{d}B(t), \quad t \geq 0, k = 1, 2, \ldots, l, \tag{1}$$

where $a_{kh}(t)$ represents time-varying coupling strength and H_{kh} stands for coupling function. SCTVCS are a class of important stochastic coupled systems. During the past few decades, the research of coupled systems has received considerable interest, since they have come to play an important role in mechanics, electrical engineering, and biological systems [1–8]. It is known that many practical applications are built on the prerequisite that coupled systems should be stable, so the stability analysis of coupled systems has been a subject of intense activities. Specific work is as follows: in [9,10], Li et al. explored the global stability of general coupled systems of ordinary differential equations by graph theory. In [11], Du and Li studied the impact of network connectivity on the synchronization and global dynamics of coupled systems of differential equations. In [12], Chen et al. investigated the stability of coupled systems with time delay. In [13], Su et al. investigated global stability of discrete–time coupled systems on networks and its application. In [14], Zhang et al. studied the stability of multi-group

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http://dx.doi.org/10.1016/j.amc.2016.06.029

models with dispersal by using graph-theoretic approach. In [15], Chen et al. investigated exponential stability for interval neural networks with discrete and distribute delays. In addition, stochastic coupled systems [16–21], discrete systems [22–25] and some kinds of delay coupled systems [26–34] were also studied widely.

Different from previous results which are all about coupled systems with constant coupling structure (i.e., $a_{kh}(t) = a_{kh}$), in practical problems, however, the coupling structure of coupled systems is not constant. Coupled systems with time-varying coupling structure (CSTVS) can be more reasonable to describe some phenomena in the real life. Therefore, the study about CSTVS is of great significance. Moreover, random disturbance and time delays are inevitable in the real world. Therefore, SCTVCS are investigated in this paper. Unfortunately, to our knowledge, few researchers investigate the stability for SCTVCS in existing literature. Our work fills this gap.

In fact, the coupling structure of coupled systems always changes over time, such as the weight of state transfer among coupled oscillators in mechanical systems, the degree of acquaintance among each other in social networks, the Internet bandwidth in the Internet. Moreover, some phenomena can be described better by coupled systems with time-varying structure (CSTVS). For example, in biomathematics, the dispersal rate among species lived in different patches always changes over time; in epidemiology, the migration of migratory birds from all over the world may bring some infectious diseases, then the transmission rate of infectious diseases will increase with a sea of migratory birds migrating. Furthermore, considering the existences of random disturbance and time delays, the investigation of SCTVCS is of great significance and worthy to study further.

Motivated by the above discussions, in this paper, by introducing time-varying coupling structure, we present a class of novel stochastic coupled systems in the form of system (1). Based on graph theory and Lyapunov method, a systematic method is given to construct an appropriate Lyapunov function for system (1), and some pth moment exponential stability (ME-stability) criteria are provided, including a Lyapunov-type theorem, and several sufficient criteria. Furthermore, the theoretical results are applied to stochastic coupled oscillators with time-varying coupling structure, two stability criteria are obtained correspondingly. Finally, two numerical examples are given to verify the effectiveness of theoretical results.

Compared with previous results [10,14,16], the chief contributions are two-fold. First, we generalize coupled systems with constant coupling structure to CSTVS, and construct an appropriate Lyapunov function for SCTVCS. Second, we first investigate the stability for a class of novel stochastic coupled systems in the form of system (1), and we obtain our results by combining graph theory with Lyapunov method, which are the generalization of previous results [10,16].

This paper is organized as follows. In Section 2, we prepare some notions and definitions to be used. Some sufficient criteria of the ME-stability for system (1) are presented in Section 3. The Section 4 is devoted to stability results for stochastic coupled oscillators with time-varying coupling structure. Finally, two numerical examples are given to show the effectiveness of the theoretical results.

2. Preliminaries

We introduce some notions first. Let $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ be a complete probability space with a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions, and $B(t)$ is a one-dimensional Brownian motion defined on the space. The mathematical expectation with respect to the given probability measure $\mathbb{P}$ is denoted by $\mathbb{E}(\cdot)$. Write $|\cdot|$ for the Euclidean norm for vectors or the trace norm of matrices. $\mathbb{R}^n$ stands for n-dimensional Euclidean space. The superscript "T" stands for the transpose of a vector or matrix. Let $C([-\tau, 0]; \mathbb{R}^n)$ be the space of continuous functions $x : [-\tau, 0] \to \mathbb{R}^n$ with norm $\|x\| = \sup_{-\tau\leq u\leq 0} |x(u)|$. $L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ stands for the family of $\mathcal{F}_0$-measurable $C([-\tau, 0]; \mathbb{R}^n)$-value random variables y such that $\mathbb{E}\parallel y\parallel^2 < \infty$. The notions $\mathbb{L} = \{1, 2, ..., l\}, \mathbb{Z}^+ = \{1, 2, ...\}, \mathbb{R}^1_+ = [0, +\infty)$ and $m = \sum_{i=1}^{l} m_i$ for $m_i \in \mathbb{Z}^+$ are used. And $C^{2,1}(\mathbb{R}^n \times \mathbb{R}^1_+; \mathbb{R}^1_+)$ represents for the family of all nonnegative functions $V(x, t)$ on $\mathbb{R}^n \times \mathbb{R}^1_+$ which are continuously twice differentiable in x and once in t.

Some essential concepts and a lemma about graph theory can be found in [10]. A digraph $\mathcal{G} = (\mathbb{L}, E)$ contains a set $\mathbb{L}$ of vertices and a set E of arcs (i, j) leading from initial vertex i to terminal vertex j. A subgraph $\mathcal{H}$ of $\mathcal{G}$ is said to be spanning if $\mathcal{H}$ and $\mathcal{G}$ have the same vertex set. A digraph $\mathcal{G}$ is weighted if each arc (i, j) is assigned a positive weight a_{ij}. Here $a_{ij} > 0$ if and only if there exists an arc from vertex j to vertex i in $\mathcal{G}$, and we call $A = (a_{ij})_{l\times l}$ as the weight matrix. The weight $W(\mathcal{G})$ of $\mathcal{G}$ is the product of the weights on all its arcs. A directed path $\mathcal{P}$ in $\mathcal{G}$ is a subgraph with distinct vertices $\{i_1, i_2, ..., i_s\}$ such that its set of arcs is $\{(i_k, i_{k+1}) : k = 1, 2, ..., s-1\}$. If $i_s = i_1$, we call $\mathcal{P}$ a directed cycle. A tree $\mathcal{T}$ is rooted at vertex i, called the root, if i is not a terminal vertex of any arcs, and each of the remaining vertices is a terminal vertex of exactly one arc. A subgraph $\mathcal{Q}$ is unicyclic if it is a disjoint union of rooted trees whose roots form a directed cycle. A digraph $\mathcal{G}$ is strongly connected if for any pair of distinct vertices, there exists a directed path from one to the other. Denote the digraph with weight matrix A as $(\mathcal{G}, A)$. A weighted digraph $(\mathcal{G}, A)$ is said to be balanced if $W(\mathcal{C}) = W(-\mathcal{C})$ for all directed cycles $\mathcal{C}$. Here $-\mathcal{C}$ denotes the reverse of $\mathcal{C}$ and is constructed by reversing the direction of all arcs in $\mathcal{C}$. For a unicyclic graph $\mathcal{Q}$ with cycle $\mathcal{C}_\mathcal{Q}$, let $\tilde{\mathcal{Q}}$ be the unicyclic graph obtained by replacing $\mathcal{C}_\mathcal{Q}$ with $-\mathcal{C}_\mathcal{Q}$. Suppose that $(\mathcal{G}, A)$ is balanced, then $W(\mathcal{Q}) = W(\tilde{\mathcal{Q}})$. The Laplacian matrix of $(\mathcal{G}, A)$ is defined as $L = (p_{kh})_{l\times l}$, where $p_{kh} = -a_{kh}$ for $k \neq h$ and $p_{kh} = \sum_{j\neq k} a_{kj}$ for $k = h$.