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A two-dimensional Haar wavelets method for solving systems of PDEs

Somayeh Arbabi[a,*], Akbar Nazari[a], Mohammad Taghi Darvishi[b]

[a] Department of Mathematics, Shahid Bahonar University, Kerman 14111, Iran
[b] Department of Mathematics, Razi University, Kermanshah 67149, Iran

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ABSTRACT

In this paper, we modify the idea of Haar wavelets method to obtain semi-analytical solutions for the systems of three-dimensional nonlinear partial differential equations. Theoretical considerations are discussed. To demonstrate the efficiency of the method, a test problem is presented. The approximate solutions of the system of three-dimensional nonlinear partial differential equations are compared with the exact solutions as well as existing numerical solutions found in the literature. The numerical solutions which are obtained using the suggested method show that numerical solutions are in a very good coincidence with the exact solutions.

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1. Introduction

Haar wavelets are the most simple ones which are defined by an analytical expression. Due to their simplicity, the Haar wavelets are very effective tools for approximating solutions of ordinary and partial differential equations. Recently, the Haar wavelet is used as a mathematical tool for solving many class of problems in physics, biology, chemical reactions and fluid mechanics. This method consists of reducing the problem to a set of algebraic equations by expanding the term which has maximum derivative, given in the equation as Haar functions with unknown coefficients. The operational matrix of integration and product operational matrix are utilized to evaluate the coefficients of Haar functions. One may find some applications of wavelets in different areas. Hariharan [1] introduced a Haar wavelet method for solving Klein–Gordon equation and sine–Gordon equation. He compared his obtained results with the exact solutions as well as the results which are obtained by modified variational iteration method (MVIM). Further, Hariharan [2] developed an accurate and efficient Haar wavelet method for solving well-known Cahn–Allen equation. By comparison between exact and Haar solutions of the Cahn–Allen equation, he showed that the numerical results are in a good agreement with the exact solutions. Also, Hariharan et al. [3] introduced a Haar wavelet method for solving Fisher's equation. Aziz et al. [4] presented a new numerical method based on Haar wavelet to solve two-dimensional nonlinear Fredholm, Volterra and Volterra–Fredholm integral equations of the first and the second kinds. They compared their numerical results with results of a few existing methods which are reported in the literature and they showed that superiority of the new method in terms of fast convergence and better accuracy. Islam et al. [5] formulated a new method based on univalent Haar wavelets for numerical solution of nonlinear integral and integro-differential equations of the first and higher orders. Aziz and Islam [6] presented two algorithms for the numerical solution of nonlinear Fredholm and Volterra integral equations using Haar wavelets. The first algorithm is

* Corresponding author.
E-mail address: s.arbabi1579@yahoo.com (S. Arbabi).

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proposed for the numerical solution of nonlinear Fredholm integral equations of the second kind, and the second one for the numerical solution of nonlinear Volterra integral equations of the second kind. These methods are designed to exploit the special characteristics of Haar wavelets in both one and two dimensions.

Bujurke et al. [7] applied the Haar wavelet series for solving the Sturm–Liouville eigenvalue problems (SLEPs). Babolian and Shahsavaran [8] using Haar wavelets obtained numerical solutions for the Fredholm integral equations. Bujurke et al. [9] used the novel single-term Haar wavelet series (STHWS) method to obtain some numerical solutions for the nonlinear, autonomous or nonautonomous, and other nonlinear oscillator equations. Shi and Cao [10] presented a computational method for solving eigenvalue problems of high-order ordinary differential equations based on the use of Haar wavelets. Çelik [11] used the Haar wavelets approximation method based on approximating the truncated double Haar wavelets series to obtain magnetohydrodynamic flow equations in a rectangular duct in presence of transverse external oblique magnetic field. Kumar and Pandit [12] presented a composite numerical scheme based on finite difference and Haar wavelets for solving time dependent coupled Burgers equation with appropriate initial and boundary conditions. Patra and Ray [13] presented an efficient wavelet operational method based on Haar wavelets to obtain the numerical solution of neutron point kinetic equation which appeared in nuclear reactor with time-dependent and independent reactivity function. Ray and Gupta [14,15] applied the wavelet collocation method to compute the numerical solutions of nonlinear partial differential equations such as Huxley and Burgers–Huxley equations. They compared the numerical solutions of the Huxley and Burgers–Huxley equations with the approximate solutions obtained by using variational iteration method as well as with the exact solutions.

Islam et al. [16] showed that a quadrature rule based on uniform Haar wavelets and hybrid functions can be easily extended to find numerical approximations for double, triple and improper integrals. Lin and Zhou [17] proposed the wavelet interpolation based on approximations for the numerical solutions of Burgers equation. Li and Zhao [18] presented the Haar wavelet operational matrix of fractional order integration and used it to solve the fractional order differential equations including the Bagley–Torvik, Ricatti and composite fractional oscillation equations. They showed that the Haar wavelet operational matrix is robust and easy to use. Ray [19] used the operational matrix of Haar wavelet method for solving Bagley–Torvik equation. In their scheme, the equation is converted to an algebraic matrix equation. Besides Haar wavelets are selected because one can compute them easily.

Ray and Patra [20] proposed an efficient numerical method to solve nonlinear damped Van der Pol equation based on the Haar wavelets. Haar wavelet collocation method is an efficient and powerful tool for solving a wide class of stationary neutron transport equation in a homogeneous isotropic medium [21]. In [22] the numerical solution for the fractional order Fokker–Planck equation has been presented using two dimensional Haar wavelet collocation method. An efficient numerical scheme based on the Haar wavelet method is applied to find numerical solution of nonlinear third-order modified Korteweg–de Vries (mKdV) equation as well as modified Burgers equations by Ray and Gupta [23]. Based on Haar wavelets, semi-analytical solution of Hunter–Saxton equation is presented by Arbabi et al. [27]. Also, Haar wavelet method is applied to compute the numerical solutions of the foam drainage equation in [28].

In this paper, we extend the Haar wavelets method to solve three-dimensional systems of nonlinear partial differential equations. We apply the extended method to solve the following system [24]

$$\begin{cases} u_t - v\,u_x - v_t\,u_y = 1 - x + y + t \\ v_t - u\,v_x - u_t\,v_y = x - y - t + 1 \end{cases} \tag{1}$$

with the following initial and boundary conditions

$$\begin{aligned} &u(x,y,0) = x + y - 1, \quad u(0,y,t) = y + t - 1, \quad u(x,0,t) = x + t - 1, \quad u(0,0,t) = t - 1 \\ &v(x,y,0) = x - y + 1, \quad v(0,y,t) = 1 - y - t, \quad v(x,0,t) = x - t + 1, \quad v(0,0,t) = -t + 1. \end{aligned} \tag{2}$$

There are some efforts to solve systems of nonlinear partial differential equations. Biazar and Eslami [24] developed a new homotopy perturbation method (NHPM) to obtain solutions of systems of nonlinear partial differential equations. Matinfar et al. [25] proposed a new homotopy analysis method to obtain solutions of three-dimensional systems of nonlinear partial differential equations. In [26] reduced differential transform method (RDTM) is applied on three-dimensional systems of nonlinear partial differential equations.

The organization of this paper is as follows: In Section 2, Haar wavelets and their integrals are described. Formulas for calculating the two-dimensional Haar wavelets method are presented in Section 3. Section 4 describes the quasilinearization technique for nonlinear terms. In Section 5 we present the convergence analysis of two-dimensional Haar wavelets method. In Section 6, we apply the new method for solving a three-dimensional system of nonlinear partial differential equations. Error analysis and numerical results are reported in Section 7. A stability computation is presented in Section 8. The paper is concluded in Section 9.

2. Haar wavelets

In 1910, the Hungarian mathematician, Alfred Haar, introduced Haar functions. In fact, the Haar function is the Daubechies wavelet of order 1. Haar wavelets are the simplest ones among different types of wavelets. These wavelets are step functions over the real line and take only three values: $-1, 0$ and 1. Due to its simplicity, the Haar wavelet is an effective tool to solve many problems arising in many areas of sciences. Usually the Haar wavelets are defined for the interval x

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