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Optimal impulsive harvesting policies for single-species populations

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ABSTRACT

In this paper, we address an optimal impulsive control problem with the aim of optimizing the harvesting rate for a non-autonomous logistic model. In addressing this problem, we combine bang–bang and singular controls for an impulsive optimization problem in which the singular controls are blocked at certain harvesting times. In our approach, we first obtain the singular controls by applying a Pontryagin optimal control framework for impulsive systems. Second, we investigate and derive a number of relationships characterizing the optimal harvesting rates and present an optimization principle: the optimal path lies as close as possible to the singular path. Finally, based on this optimization principle, we obtain analytical expressions for the optimal harvesting policy for our problem and use numerical simulations to demonstrate the effectiveness of our method. This study develops concepts and theory related to continuous control problems and applies them to impulsive control problems, and extends the related work of Xiao et al. (2006) and Braverman and Mamdani (2008).

1. Introduction

Optimal population harvesting and resource management present some of the most fundamental challenges in population ecology [1]. Recently, continuous and impulsive optimal harvesting problems have attracted significant attention [2–9]. Suppose that the population size is governed by the following non-autonomous differential equation:

$$\frac{dN}{dt} = f(t, N(t)), \tag{1}$$

where $f(t, N)$ describes the growth rate. The model in (1) has been frequently studied in the context of continuous harvesting (cf. [1,5,7]). However, as mentioned by Xiao et al. [3] and Zhang et al. [6], in many cases impulsive harvesting is more realistic than continuous harvesting. Impulsive harvesting at fixed times t_i $(i \in \{1, 2, \dots\})$, can be either proportional to the size of the population, in the form

$$N(t_i^+) = (1 - E_i)N(t_i), \tag{2}$$

or constant, taking the form

$$N(t_i^+) = N(t_i) - h_i, \tag{3}$$

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where $N(t_i^+) = \lim_{h\to 0^+} N(t_i + h)$. The optimal strategies for the cases described by both (2) and (3) for the logistic equation have been widely studied [3,6,10–12], and most of the optimal control problems mentioned above choose either the harvesting times or the harvesting efforts as the control variables. Refs. [13–15], on the other hand, consider a class of more complex optimization harvesting problems for a dynamic model arising in aquaculture operations, in which both the harvesting times and the harvesting efforts are optimized to maximize the total revenue. We refer the reader to [16–20] for a comprehensive review and discussion of the optimization of the impulsive model.

1.1. Problem statement

In optimal control problems in which the control variable appears to be linear, the optimal control is a concatenation of bang–bang and singular arcs. In many problems, e.g. in the fishing problem of Clark [1], the singular control represents the most important part of the control strategy. Because of the control constraints, a control policy does not always follow the singular control. For continuous optimization problems, Arrow [21,22] introduced the term 'blocked interval', which refers to any time interval during which the singular control is blocked. It is worth noting that similar blocked phenomena can also occur in impulsive optimization problems.

To the best of our knowledge, almost all known analytical results regarding impulsive optimal control problems in which the control variable appears to be linear only consider the case where optimality is attained at each non-harvested period. This requires that the singular control be feasible; therefore, additional hypotheses are necessary to ensure the feasibility of the singular control (cf. [3,17,20,23]). However, the control constraints may block singular controls at certain harvesting times; such a control is called a 'blocked singular control'. Blocked singular controls for impulsive optimization problems have not yet been investigated in existing literatures, and an open problem proposed in [11,17] is to determine the optimal control strategy for the case where optimality cannot be attained at some non-harvested periods, that is, to find the optimal control strategy when the singular control is blocked.

For an optimal impulsive control problem in which the control variable appears to be linear, if the singular controls are blocked at certain harvesting times, then the optimal control must be a combination of bang–bang and singular controls. However, it is a complicated and difficult task to determine the correct juxtaposition of these elements. By investigating a number of relationships characterizing the optimal harvesting rates, we investigate and solve a blocked singular control problem that is encountered in our impulsive optimization control problem.

Let us now consider the following logistic model with impulsive harvesting in the variable environment:

$$\begin{cases} \frac{dN}{dt} = r(t)N(t)\left[1 - \frac{N(t)}{K(t)}\right], & t \neq t_i, \quad i \in I, \\ N(t_i^+) = (1 - E_i)N(t_i), & i \in I, \\ N(t_0) = N_0. \end{cases} \tag{4}$$

The logistic system (4) with a variable growth rate and carrying capacity $K(t)$ between impulses is subject to harvesting at times t_i, $i \in I = \{0, 1, \ldots, n\}$, $t_0 < t_1 < \cdots < t_n < T$.

We choose the harvesting efforts $E_i\,(i \in I)$ as the control variables, with the constraints

$$0 \le E_i \le 1, \quad i \in I. \tag{5}$$

Let us maximize the total harvesting yield over the interval $[t_0, T]$ as follows:

$$J = \sum_{i=0}^{n} E_i N(t_i) + N(T). \tag{6}$$

For the impulsive optimization problem defined by (4)–(6), we investigate the optimal control policy for any given initial population, growth rate, and set of harvesting times.

1.2. Our contributions

(1) We investigate an open problem, that of the blocked singular control strategy for impulsive control problems (see Refs. [11,17]). We solve this problem for a non-autonomous logistic model and obtain the explicit expression of the optimal policy for any given initial population, growth rate, and set of harvesting times.

(2) We develop concepts and theory related to continuous control problems and apply them to impulsive control problems. The blocked singular control for 'continuous' control problems has been discussed in [1]. There, the optimal control is a concatenation of rather small bang–bang arcs and a large singular arc (turnpike control). However, the blocked singular control strategy for impulsive control problems has not yet been studied. To solve this problem, inspired by the methodology for the continuous control problem, we first prove a similar optimization principle: the optimal path lies as close as possible to the singular path. Then, based on this principle, we go on to apply the 'premature switching principle' to the impulsive control problem, thereby deriving a number of relationships characterizing the optimal harvesting rates.