Applied Mathematics and Computation 292 (2017) 210–227

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Bifurcations in a delayed fractional complex-valued neural network

Chengdai Huang[a], Jinde Cao[a,b,*], Min Xiao[c], Ahmed Alsaedi[d], Tasawar Hayat[b,e]

[a] Research Center for Complex Systems and Network Sciences, and Department of Mathematics, Southeast University, Nanjing 210096, China
[b] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] College of Automation, Nanjing University of Posts and Telecommunications, Nanjing 210003, China
[d] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[e] Department of Mathematics, Quaid-I-Azam University, Islamabad 44000, Pakistan

ARTICLE INFO

Keywords:
Time delays
Stability
Hopf bifurcation
Fractional neural networks
Complex-valued

ABSTRACT

Complex-valued neural networks (CVNNs) with integer-order have attracted much attention, and which have been well discussed. Fractional complex-valued neural networks (FCVNNs) are more suitable to describe the dynamical properties of neural networks, but have rarely been studied. It is the first time that the stability and bifurcation of a class of delayed FCVNN is investigated in this paper. The activation function can be expressed by separating into its real and imaginary parts. By using time delay as the bifurcation parameter, the dynamical behaviors that including local asymptotical stability and Hopf bifurcation are discussed, the conditions of emergence of bifurcation are obtained. Furthermore, it reveals that the onset of the bifurcation point can be delayed as the order increases. Finally, an illustrative example is provided to verify the correctness of the obtained theoretical results.

1. Introduction

In the past decades, the qualitative analysis of neural networks (NNS) has received considerable attention due to its successful applications in various fields of science and engineering such as image processing, combinatorial optimization [1,2], associative memories [3], pattern recognition, signal processing, control engineering [4] and so on. Wen et al. found that the global pinning synchronization in the neural networks can be achieved if some nodes are appropriately pinned and the coupling is carefully selected [4]. Due to the finite switching speed of amplifiers, time delay inevitably exists in neural networks. On one hand, time delays are harmful to the dynamical behaviors of considered neural networks causing oscillation, poor performance. On the other hand, a proper time delay is advantageous for dynamic NNS, such as, it can improve the stability performance of networks [5]. Thus, the consideration of time delays is more and more significant in the study of stability analysis of neural networks. Moreover, many practical applications of NNS are heavily dependent on the dynamical properties of NNS. The analysis of dynamical behaviors of NNS is necessary and important in the design of

* Corresponding author at: Research Center for Complex Systems and Network Sciences, and Department of Mathematics, Southeast University, Nanjing 210096, China. Fax: +862552090590.
E-mail address: jdcao@seu.edu.cn (J. Cao).

http://dx.doi.org/10.1016/j.amc.2016.07.029
0096-3003/

dynamical neural networks. Therefore, many researchers have elaborately investigated the dynamical properties of NNS time delays including stability, bifurcation and synchronization, and some remarkable results have been reported [6–10].

Fractional calculus has become a hot topic in recent years and many applications are spotted in the field of physics and engineering [11,12]. In [12], a modified Kalman filter algorithm for discrete linear fractional order system with Lévy noise has been developed, and the obtained novel FKF can estimate the system states under non-Gaussian Lévy noises efficiently. Fractional calculus can be regarded as a generalization of the classic integer-order differential and integral calculus, which mainly deals with differentials and integrals of an arbitrary order. A great number of the real world objects are generally identified and described by the fractional model. Fractional model is more accurate compared with the integer-order model. The main advantage of fractional model in comparison with integer-order model is that a fractional derivative provides an excellent tool in the description of memory and hereditary properties of various processes. In recent years, fractional calculus has been integrated into neural networks, mainly because it can represent infinite memory. In [13], the authors revealed that fractional differentiation provides neurons with a fundamental and general computation ability that can contribute to efficient in formation processing, stimulus anticipation and frequency-independent phase shifts of oscillatory neuronal firing. Thus, it is an extraordinarily important improvement for the incorporation of a memory term into a NN model. More recently, there has been an increasing interest in the investigation of fractional NNs, and some important and interesting results were obtained [14–16]. In [15], global stability results of delayed fractional Hopfield NNs with hub structure and ring structure were derived, respectively. In [16], the authors considered the issue of finite-time stability for a class of fractional NNs with time delays, and delay-dependent sufficient conditions ensuring stability of such fractional NNs over a finite-time interval was obtained.

In NNs, the complex-valued neural network (CVNN) is one of very essential and valuable NNs, its states, connection weights and activation functions are expressed in terms of complex-valued data, and it processes information on the complex plane. CVNNs are a fast growing field of research in both theoretical and application [17–20]. In [17], the complex-valued neural networks were applied to optical signal processing via phase-sensitive detection schemes, the author found that the summation and nonlinear transform of the complex signals are processed quickly in parallel by using an optical circuit for proposed system. In [20], the problem of multistability for an n-dimensional CVRNN with a class of real–imaginary-type activation functions was studied, the proposed CVRNN models can store many more memories and patterns. It revealed that CVNNs can provide higher recognition performance compared with classical real-valued NNs [21]. By using complex values in the neural networks, the amplitude information as well as phase information of the signal can be treated so that realization of new types of information processing not yet possible in the conventional real-valued model becomes achievable [22]. Therefore, the dynamical properties of complex-valued NNs received attention in recent years, various results have been derived [23–25]. It is pity that these results do not introduce the fractional calculus into the CCNNs. In fact, fractional CVNNs can describe more precisely the dynamics of NNs in comparison with classical integer-order CVNNs see [26,27]. In [27], the problem of existence and uniform stability analysis of fractional CVNNs with time delays was considered, sufficient condition for the existence and uniform stability analysis of such networks was derived.

It is well known that Hopf bifurcation analysis for a nonlinear system is a very effective approach [28–30], not only involving a discussion of stability properties, but also consisting of analysis of others dynamic behaviors, such as periodic oscillation, bifurcation and chaos, from which one can obtain more information about periodic solution properties near stationary state of systems. As is known to all, Hopf bifurcation in inter-order system have been extensively and sufficiently studied, and a large number of valuable results have been reported. Recently, Hopf bifurcation of delayed fractional systems have attracted increasing interest [31–33]. In [31], the authors designed a linear time-delayed feedback controller to control the delayed fractional small world network, it was found that the onset of Hopf bifurcation can be delayed by carefully selected the gain and the order, respectively. In [33], the issue of bifurcation control for a novel delayed fractional gene regulatory network was discussed, and it revealed that the proposed hybrid controller can successfully stabilize Hopf bifurcation and achieve some desirable dynamics. However, to the best of our knowledge, the problem of stability and bifurcation for fractional complex-valued neural networks (FCVNNs) with time delay is a challenging problem, which is still unsolved. We are devoted to dealing with the interesting and important problem in this paper.

The main contributions of this paper are listed as follows: (1) We generalize an integer-order NN to the version of FCVNN. (2) The present paper is one of the first that attempt to study the stability and bifurcation in a delayed FCVNN by using time delay as the bifurcation parameter, and the conditions of occurrence of Hopf bifurcation for such FCVNN are established. (3) The effects of the order on the bifurcation point is discussed for the proposed FCVNN.

The outline of the paper is highlighted as follows: In Section 2, some definitions are given and the method of stability analysis for the linearized delayed fractional systems are recalled. In Section 3, the statement of problem are clarified. The conditions of Hopf bifurcation are established in Section 4. Numerical simulations are carried out via a numerical example in Section 5. Conclusions are presented in Section 6.

2. Preliminaries

There are several definitions of fractional derivatives. The Riemann–Liouville definition and the Caputo definition are commonly used. Since the Caputo derivative only requires initial conditions given by means of integer-order derivative, representing well-understood features of physical situation and making it more applicable to real world problems. This paper is based on the Caputo derivative.