Dynamics analysis and control optimization of a pest management predator–prey model with an integrated control strategy

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ABSTRACT

Pest management is a complex issue in real applications, and a practical program in pest control in general involves two pest thresholds, where the biological control and chemical control are activated respectively. Aiming at providing a good balance between the biological control and chemical control, this work presented an integrated pest management predator–prey model, where the yield of releases of predator and the strength of pesticide spraying are linearly dependent on the selected control level. Firstly, to determine the frequency of spraying chemical pesticide and releasing of predator, the existence of the order-1 periodic orbit of the proposed model is discussed by the successor function method. And then, to ensure a certain robustness of adopted control, the stability of the order-1 periodic orbit is verified by a stability criterion extracted for a general semi-continuous dynamical system. In addition, to minimize the total cost (i.e. culturing predators and spraying pesticide) in pest control, an optimization problem is formulated and the optimum pest control level is obtained. At last, to complement the theoretical results, the numerical simulations with a specific model are carried out step by step.

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1. Introduction

Threat of pests to agricultural productions is a serious problem across the world, which makes pests control being an interesting topic and attracts a great attentions on the development of effective pest control methods. Integrated pest management (IPM, involves combining biological, mechanical, and chemical tactics) is a very effective method in controlling pests with minimal use of harmful pesticides and other undesirable measures, which has been proved to be more effective than the classic methods both experimentally (e.g. [1–3]) and theoretically (e.g. [4,5]). The goal of IPM is not to eradicate pests, instead to control the number of the pests under an economic threshold (ET) and protect ecosystems in maximum extent.

IPM strategy may cause biological population change radically due to the variety of manual intervention, and impulsive differential equations (IDEs) in mathematics becomes a powerful tool to describe these phenomena [6]. Theoretical studies on the theory of IDEs can be found in [7–10]. Based on the theoretical results, many scholars have introduced impulsive

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differential equations in predator–prey system to model the pest control, for example: the periodic release of predators [11–16]; the periodic release of pests infected by a disease [17,18]; the periodic release of predators and infected pests [19]; the periodic release of infected pests combined with periodic applications of pesticides [20]; the periodic release of predators, pests combined with periodic applications of pesticides [21–24] and state dependent release of predators combined with applications of pesticides [3,25–35]. As a pioneer work in state-dependent pest control, Tang et al. [3,25,26] introduced the following predator–prey model with state-dependent impulsion, i.e.

$$
\begin{cases}
\begin{rcases}
\dfrac{dx(t)}{dt} = x(t)(b_1 - a_{11}x(t)) - a_{12}x(t)y(t) \\
\dfrac{dy(t)}{dt} = y(t)(-b_2 + f_{\mathrm{conv}}(x(t)))
\end{rcases} & x \neq ET \\
\begin{rcases}
\Delta x = -px(t) \\
\Delta y = -qy(t) + \tau
\end{rcases} & x = ET
\end{cases}
\tag{1.1}
$$

where f_{conv} describes the per capita conversion rate form prey to predator, ET is the pest economic threshold and $0 < p < 1$, $0 \leq q < 1$ and $\tau > 0$. Notice that the model (1.1) assumes that the measures of releasing the predator and spraying pesticide are taken at the same pest level, and also at the same time. But in real applications, the two control measures may be adopted at different pest levels. Motivated by this situation, Nie et al. [36,37], Tian et al. [38,39], Zhao et al. [40] and Zhang et al. [41] proposed and analyzed the following predator–prey system model by assuming that releasing natural enemies and spraying pesticide are taken at different thresholds, i.e.

$$
\begin{cases}
\begin{rcases}
\dfrac{dx(t)}{dt} = x(t)(g_{\mathrm{grow}}(x(t)) - a_{12}y(t)) \\
\dfrac{dy(t)}{dt} = y(t)(-b_2 + f_{\mathrm{conv}}(x(t)))
\end{rcases} & \begin{array}{l} x \neq SHT, x \neq SHT \\ \text{or } x = SHT, y > y_{ML} \end{array} \\
\begin{rcases}
\Delta x = 0 \\
\Delta y = \tau_{\max}
\end{rcases} & x = SHT, y \leqslant y_{ML} \\
\begin{rcases}
\Delta x = -px(t) \\
\Delta y = -qy(t) + \tau_{\min}
\end{rcases} & x = EIT
\end{cases}
\tag{1.2}
$$

with $f_{\mathrm{conv}}(x(t))$ described by $a_{21}x(t)$ and $\lambda a_{12}x(t)/(1 + a_{12}hx(t))$, and $g_{\mathrm{grow}}(x(t))$ described by $b_1 - a_{11}x(t)$ and $\ln(K/x(t))$ respectively, where g_{grow} describes per capita growth rate of the pest in absent of predator, and y_{ML} represents the predator maintainable level at $x = SHT$, which is defined by $y_{ML} \triangleq a_{12}^{-1} g_{\mathrm{grow}}(f_{\mathrm{conv}}^{-1}(b_2))\sigma_{SHT}$, where $\sigma_{SHT} \in [0, g_{\mathrm{grow}}(SHT)/g_{\mathrm{grow}}(f_{\mathrm{conv}}^{-1}(b_2))]$ is a reference parameter and f_{conv}^{-1} is the inverse function of f_{conv}.

The idea of involving the biological and chemical controls at different pest thresholds is interesting and has practical significance. But in this process a key problem should be pointed out, that is the biological control is adopted when the pest density x reaches the threshold SHT while the predator density y is lower than its maintainable level y_{ML}, but for a higher pest density $x = ET$, where $SHT < ET < EIT$, there is no control strategy adopted. This is obviously unreasonable. In addition, from economic and practical point of view, the control taken at SHT sometimes seems to be a little early and the yield of releases of the predator will be also large, while the control taken at EIT seems to be a little late and the chemical control strength will be also high. This motivated us to consider a practical problem: if we select one pest level between SHT and EIT to take control, which level is the best? How to keep a good balance between yield of releases of the predator and strength of pesticide spraying in practice? Based on this consideration, this work presents a pest management predator–prey model by integrating the biological and chemical control strategies at SHT and EIT respectively.

This paper is organized as follows. In Section 2, a pest management predator–prey model by an integrated control strategy is put forward, where the yield releases of predator and strength of pesticide spraying are linearly dependent on the selected control level. In Section 3, the existence of the order-1 periodic orbit is discussed by successor function method. The stability of the order-1 periodic orbit is verified by a stability criterion extracted for a general semi-continuous dynamic system. In addition, an optimization problem is formulated to minimize the total cost in pest control. In Section 4, the numerical simulations are carried out with a specific model to complement the theoretical results step by step. The final conclusion is presented in Section 5.

2. Model formulation

Let $x(t)$ and $y(t)$ denote the prey and predator densities at time t. The pest per capita growth rate g_{grow} in absent of predator is assumed to follow Ludwig's model [42]

$$
g_{\mathrm{grow}}(x(t)) = r\left(1 - \frac{x(t)}{K}\right),
$$

where r is the birth rate, K is the environmental carrying capacity for the prey in absent of predator. For the species without environmental carrying capacity constraint, K can be chosen as a larger positive constant. The predation is assumed to be