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The dynamics of an impulsive predator–prey model with communicable disease in the prey species only

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ABSTRACT

In this paper, we propose an impulsive predator–prey model with communicable disease in the prey species only and investigate its interesting biological dynamics. By the Floquet theory of impulsive differential equation and small amplitude perturbation skills, we have deduced the sufficient conditions for the globally asymptotical stability of the semi-trivial periodic solution and the permanence of the proposed model. We also give the existences of the “infection-free” periodic solution and the “predator-free” solution. Finally, numerical results validate the effectiveness of theoretical analysis for the proposed model in this paper.

1. Introduction

Biological control is an important component of integrated pest management programmes, and it relies on parasitism, predation, herbivore, or other natural mechanisms. However, it typically also contains active human management activities. Natural enemies play important roles in reducing the numbers of mites and pest insects. Moreover, biological control has a great benefit that is its safety for human health and the environment. There are some successful biological control examples which include uses of the predatory mites Phytoseiulus persimilis and Neoseiulus californicus against the red spider mite Tranychus urticae Koch in field-grown strawberries [1] and the use of the predatory arthropod Orius sauteri against the pest Thrips palmi Karny to protect eggplant crops in greenhouses [2]. Beauveria bassiana is often used to manage many kinds of insect pests including thrips, aphids, whiteflies and weevils. Because bacteria used for biological control infect insects by their digestive tracts, insects with sucking mouth parts like aphids and scale insects are very difficult to control with bacterial biological control [3–7]. In addition, bacillus thuringiensis is generally the most widely applied species of bacteria mainly used for biological control, with at least four sub-species used to control Coleoptera (beetles) , Diptera (true flies) and Lepidoptera (moths and butterflies) [8].

In recent years, many biologists tried their best to study the management of renewable natural resources, and extended impulsive differential equations to the models in the permanence of ecosystems [9–14]. Some researchers studied that the infected pests spread disease into the healthy wild population, and used the corresponding strategy in controlling pests [15].

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Phillips et al. researched that biocontrol can have disastrous consequences when it is poorly planned [16]. Lakshmikantham detailedly introduced the theory of impulsive differential equations and highlights the significance of Floquet theory [17]. Orr detailedly gave the history and informative review of biocontrol [18]. Li et al. proposed a SIS epidemic model incorporating media coverage and analyzed the dynamics of this disease model under constant and pulse vaccination, and they confirmed their theoretical results by numerical simulations [19]. Many experts investigated the impulsive differential systems such as population ecology [20], birth pulse [21] and chemotherapeutic treatment of disease [22], and they studied the properties of the stability and periodicity of solutions of the system. Tang and Chen studied the impulsive effect on the ecological situations by investigating a classical periodic Lotka–Volterra predator–prey system with impulsive effect [23]. Liu proposed a Lotka–Volterra predator–prey model with impulsive effect at fixed moment and investigated it according to the fact of integrated pest management [24]. Stone analyzed an SIR epidemic model with pulse vaccination strategy and thought that impulsive effect was important on epidemic models [25]. Donofrio studied the pulse vaccination strategy in the SIR epidemic model and stability properties of pulse vaccination strategy in SEIR epidemic model [26,27]. Hadeler investigated the first eco-epidemiological model of predator–prey population with parasite infection and obtained that a disease was spreading among interacting populations [28]. Panetta studied the bifurcation of nontrivial periodic solutions for an impulsively perturbed system of ordinary differential equations which models an integrated pest management strategy by means of a fixed point approach [29]. Lakmeeh studied the bifurcation of nontrivial periodic solutions of impulsive differential equations arising chemotherapeutic treatment [30]. Liu et al. proposed a system of impulsive differential equations describing predator–prey dynamics with impulsive effect and analyzed the existences of "infection-free" periodic solution and the "predator-free" periodic solution via bifurcation [31]. Wang et al. proposed the pest management model with spraying microbial pesticide and releasing the infected pests, and investigated the dynamics of such a system by using the Floquet theory for impulsive differential equations [32].

Some experts have controlled the pests by exploiting viruses and simultaneously releasing the pest population [33,34]. First, a small amount of pathogens are introduced into a pest population with the expectation that it will generate an epidemic and that it will subsequently be endemic. The success of this method depends on the survival of the microbes which in turn depends on environmental factors. At the same time, we consider to release the pests infected in the laboratories to the pest population with periodic impulsive effect. The infected pests have little effect on the crops. The susceptible pests become infected through direct contact with the infective ones or through encountering the free-living infective stage in the environment. Thus it can infect the pest population and result in the death of them continuously. The main purpose of this paper therefore is to formulate and investigate an epidemiological model for the bio-control of a pest. In fact, the theoretical investigation and its application analysis can be found in almost every field [35–43]. This pest population is assumed to grow according to a logistic curve in the absence of disease [44,45]. Further, we consider the dynamics of an impulsive predator–prey model with communicable disease in the prey species only with periodic impulsive effect.

The paper is organized as follows: In Section 2, we introduce the formulation of pest control model and some definitions. We give some preliminary lemmas and theorems and obtain the conditions for the globally asymptotical stability and the permanence of the semi-trivial periodic solution in Section 3. In Section 4, we analyze the existences of the "infection-free" periodic solution and the "predator-free" solution. We present numerical simulations to illustrate our results and the effects of impulse in Section 5.

2. The formulation of pest control model

Let $N(t)$ denote the density of an original insect pest population. Herein we give an assumption: it will grow in terms of the regulation of a logistic curve with the capacity r/a and a constant intrinsic birthrate r. The dynamics of $N(t)$ is given as the following differential equation by establishing mathematical model and considering the practical value:

$$\dot{N}(t) = N(t)(r - aN(t)).$$

When a pest pathogen as biotic insecticide intrudes into the pest community, the pest species is divided into two classes: The first class is the susceptible pest whose density is represented by $S(t)$ at the time t; the second class is the infected pest whose density is denoted by $I(t)$ at the time t. So the total density of the population at any time t is

$$N(t) = S(t) + I(t).$$

We further assume that both the susceptible and infected pest individuals are capable of reproducing. The incidence is given by the simple mass action incidence with transmission coefficient $\lambda > 0$. The constant $\beta > 0$ acts as the mortality due to the illness. Thus, the insect-pathogen model yields

$$\begin{cases} \dot{S}(t) = [r - a(S(t) + I(t))]S(t) - \lambda S(t)I(t), \\ \dot{I}(t) = \lambda S(t)I(t) + (r - \beta - a(S(t) + I(t)))I(t). \end{cases}$$

If natural enemies of the pest are applied, these enemies only prey on the susceptible pest and the density of the natural enemies is represented by $Y(t)$ at the time t, then the insect-pathogen model yields