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Improved stability criteria for linear systems with interval time-varying delays: Generalized zero equalities approach

Seok Young Lee [a], Won Il Lee [b], PooGyeon Park [b,*]

[a] Division of IT Convergence Engineering, Pohang University of Science and Technology, Pohang, Republic of Korea
[b] Department of Electrical Engineering, Pohang University of Science and Technology, Pohang, Republic of Korea

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ABSTRACT

This paper suggests first-order and second-order generalized zero equalities and constructs a new flexible Lyapunov–Krasovskii functional with more state terms. Also, by applying various zero equalities, improved stability criteria of linear systems with interval time-varying delays are developed. Using Wirtinger-based integral inequality, Jensen inequality and a lower bound lemma, the time derivative of the Lyapunov–Krasovskii functional is bounded by the combinations of various state terms including not only integral terms but also their interval-normalized versions, which contributes to make the stability criteria less conservative. Numerical examples show the improved performance of the criteria in terms of maximum delay bounds.

1. Introduction

Stability analysis of time-delay systems has been one of the hottest issues since time delays occur in many dynamic systems such as networked control systems, neural network systems, mechanical systems and chemical processes. The occurrence of such time delays often causes undesirable dynamic behaviors such as severe performance degradation or even instabilities of the relevant systems [1]. Thus, stability analysis of time-delay systems has to be considered before implementing various control strategies. The field of stability analysis of time-delay systems can be classified into two categories that are delay-independent stability analysis and delay-dependent one. Currently, many researchers mainly have focused on the delay-dependent stability analysis based on various Lyapunov–Krasovskii functionals because the delay-dependent stability criteria are generally less conservative than the delay-independent ones.

The main topic of the stability analysis for time-delay systems is to find the maximum delay bounds guaranteeing the asymptotic stability of the concerned systems as large as possible. To do this, with an appropriate Lyapunov–Krasovskii functional, obtaining a precise bound of the time derivative of a Lyapunov–Krasovskii functional is crucial since its integral quadratic functions cannot be directly handled to construct linear matrix inequality (LMI) conditions. Consequently, to derive tighter bounds of the time derivatives of various Lyapunov–Krasovskii functionals, many mathematical tools have been proposed such as a slack-matrix-based integral inequality [2–4], Jensen inequality [5–17], a free-weighting matrix method [18–20], a lower bound lemma for reciprocal convexity [9,13,16,17,21], a zero equality approach [16,22] and a delay partitioning method [23,24]. Among the above methods, especially the Jensen inequality has been widely used because it could give identical performance to the slack-matrix-based integral inequality with the less number of decision variables [21]. Recently,

* Corresponding author. Tel.: +82 54 279 2238; Fax: +82 54 279 2903.
E-mail addresses: suk122@posetch.ac.kr (S.Y. Lee), wilee@posetch.ac.kr (W.I. Lee), ppg@posetch.ac.kr (P. Park).

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however, a new integral inequality, called Wirtinger-based integral inequality, that reduces the conservatism of the Jensen inequality was proposed in [25] and has attracted considerable attentions [26,27]. Further, there have been an attempt to generalize the Wirtinger-based integral inequality [28]. However, in [25–28], the Lyapunov–Krasovskii functionals can be more flexible with augmented integrands, and the utilized zero equality can be more general by handling the combinations of not only a state vector $x(t)$ and $\dot{x}(t)$ but also an integral of $x(t)$. Thus, there is still room for reducing the conservatism.

With the above observations, in this paper, the stability analysis for linear systems with interval time-varying delays is revisited. The main contributions of this paper lie in three aspects.

1. A new flexible Lyapunov–Krasovskii functional containing more state terms is proposed to make less conservative results. In some integrands of the functional, not only $x(s), \dot{x}(s)$ but also $\int_s^t x(r)dr$, $\int_s^{t-h_1} x(r)dr$, $\int_{t-h_2}^s x(r)dr$, $\int_{t-h_1}^s x(r)dr$ are included, where h_1 and h_2 are lower and upper bounds of a time-varying delay, $h(t)$, respectively. To the best of authors' knowledge, the proposed functional has not been reported yet in the stability analysis for time-delay systems.
2. Inspired by the works of Kwon et al. [16] and Kim et al. [22], generalized zero equality lemmas are proposed . Based on the proposed lemmas, it can be verified that the zero equalities in [16,22] are special cases of the proposed ones. Further, new zero equalities are derived and merged into the time derivative of the Lyapunov–Krasovskii functional to reduce the conservatism of the stability criteria.
3. The time derivative of the Lyapunov–Krasovskii functional is bounded by the combinations of various state terms including not only integral terms such as $f_i(t)$, $g_i(t)$ $(i = 1, 2, 3)$ but also their interval-normalized versions such as $\frac{1}{h(t)-h_1} f_j(t)$, $\frac{1}{h_2-h(t)} g_j(t)$ $(j = 1, 2, 3)$, where $f_1(t) = \int_{t-h(t)}^{t-h_1} x(r)dr$, $f_2(t) = \int_{t-h(t)}^{t-h_1} \int_{t-h(t)}^{s} x(r)dr\, ds$, $f_3(t) = \int_{t-h(t)}^{t-h_1} \int_{s}^{t-h_1} x(r)drds$, $g_1(t) = \int_{t-h_2}^{t-h(t)} x(r)dr$, $g_2(t) = \int_{t-h_2}^{t-h(t)} \int_{t-h_2}^{s} x(r)\, drds$, $g_3(t) = \int_{t-h_2}^{t-h(t)} \int_{s}^{t-h(t)} x(r)drds$. This approach contributes to make the stability criteria less conservative. Also, relations of the integral terms and their interval-normalized versions yield meaningful zero equality conditions with slack matrices.

Based on the contributions, improved stability criteria for linear systems with interval-time varying delays are derived as Theorem 1 and Corollary 1. Numerical examples show improved performance of the criteria in terms of maximum delay bounds.

Notations: Throughout this paper, $X > 0$ $(X \geq 0)$ means that X is a real symmetric positive definite matrix (positive semidefinite). $col\{x_1, x_2, \ldots, x_n\}$ means $[x_1^T, x_2^T, \ldots, x_n^T]^T$. $diag\{X, Y\}$ means $\begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}$. The symmetric blocks will be readily denoted by $\star$ when necessary. In addition, for any rectangular matrix M, $sym\{M\}$ denotes $M + M^T$.

2. Problem formulation and preliminaries

A linear system with a time-varying delay has been widely used to compare effectiveness of mathematical tools for less conservative stability criteria. Thus, in this paper, the following linear system with a time-varying delay is considered:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-h(t)), \ t \geq 0, \\ x(t) &= \phi(t), \ -h_2 \leq t \leq 0, \end{aligned} \tag{1}$$

where $x(t)$ is a state vector, the initial condition, $\phi(t)$, is a continuous function, a time-varying delay, $h(t)$, is a continuous function satisfying $0 \leq h_1 \leq h(t) \leq h_2$, h_1 and h_2 are constant values and $h_{12} = h_2 - h_1$.

Before deriving our main results, we introduce the following lemmas:

Lemma 1 (a lower bound lemma for reciprocal convexity [21])**.** *Let $f_1, f_2, \ldots, f_N : \mathbf{R}^m \mapsto \mathbf{R}$ have positive values in an open subset $\mathbf{D}$ of $\mathbf{R}^m$. Then, the reciprocally convex combination of f_i over $\mathbf{D}$ satisfies*

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

subject to

$$\left\{ g_{i,j} : \mathbf{R}^m \mapsto \mathbf{R}, g_{j,i}(t) = g_{i,j}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0 \right\}.$$

Lemma 2 [17]**.** *For any vectors x_1,x_2, matrices Q_i $(i = 1, \ldots, 4)$, S and real scalars $\alpha > 0$, $\beta > 0$ satisfying $\alpha + \beta = 1$, the following inequality holds:*

$$-\tfrac{1}{\alpha} x_1^T Q_1 x_1 - \tfrac{1}{\beta} x_2^T Q_2 x_2 - \tfrac{\beta}{\alpha} x_1^T Q_3 x_1 - \tfrac{\alpha}{\beta} x_2^T Q_4 x_2 \leq - \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} Q_1 & S \\ S^T & Q_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

subject to

$$0 < \begin{bmatrix} Q_1 + Q_3 & S \\ S^T & Q_2 + Q_4 \end{bmatrix}.$$