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Soliton fusion and fission in a generalized variable-coefficient fifth-order Korteweg-de Vries equation in fluids

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ABSTRACT

Under investigation in this paper is a generalized variable-coefficient fifth-order Korteweg-de Vries equation, which describes the interaction between a water wave and a floating ice cover or the gravity-capillary waves. Via the Hirota method, Bell-polynomial approach and symbolic computation, bilinear forms, N-soliton solutions, Bäcklund transformation and Lax pair are derived. Infinitely-many conservation laws are obtained based on the Bell-polynomial-typed Bäcklund transformation. Soliton fusion and fission, and influence of the variable coefficients from the equation are analyzed: Both variable coefficients $c(t)$ and $n(t)$ are in direct proportion to the soliton velocities but have no effect on the amplitudes, while another constant coefficient α can affect the types of the interactions, in the sense of the elastic or inelastic. Elastic–inelastic interactions among the three solitons are presented as well.

1. Introduction

Solitons, as one type of the solutions for nonlinear evolution equations (NLEEs), have been found to describe the nonlinear wave phenomena in such fields as fluid dynamics, plasma physics, fiber optics, condensed matter physics and chemistry [1–4]. Formation of the solitons is due to the balance between the nonlinear, dispersive and other terms in the NLEEs [5]. Analytic methods have been developed to search for the solitons, e.g., the inverse scattering transformation, Hirota method, Bell-polynomial approach and Wronskian technology [6–11].

Soliton interaction can be divided into the elastic and inelastic types [12–25]. During the elastic interaction, solitons keep their amplitudes, velocities and shapes invariant except for some phase shifts after the interactions [12]. For the inelastic interaction, the shape-changing one in a coupled NLEE [13] or the resonant one can occur under certain condition(s) [14–25]. Soliton resonance has been observed in the cold plasmas [14], fluids [18], optical fibers [22,23], nonlinear ion-acoustic systems [24] and discrete soliton systems [25]. Soliton fusion and fission are two kinds of the soliton resonance [18–21]: the phenomenon that one soliton splits up into two or more solitons is called the soliton fission, while that two or more solitons merge into one single soliton, named the soliton fusion [18,19].

Korteweg-de Vries (KdV) equation, as a model to describe the propagation of the long one-dimensional, small amplitude shallow water waves [12,26], has been extended to the higher-order and higher-dimensional versions [27–32]. Among them,

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fifth-order KdV and (2+1)-dimensional KdV extensions have been found to possess the soliton resonance [27,31,32], which are the Sawada–Kotera (SK) equation with a non-vanishing boundary conditions and the Kadomtsev–Petviashvili equation.

In this paper, with symbolic computation [33,34], we will investigate a generalized variable-coefficient fifth-order KdV equation [35],

$$u_t + a(t)uu_{xxx} + b(t)u_xu_{xx} + c(t)u^2u_x + d(t)uu_x + e(t)u_{xxx} + l(t)u_{xxxxx} + m(t)u + n(t)u_x = 0, \tag{1}$$

where u is a real function of space x and time t, and $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$, $l(t)$, $m(t)$ and $n(t)$ are the analytic functions of t. Eq. (1) can be used to describe the interaction between a water wave and a floating ice cover or the gravity-capillary waves in fluid dynamics [35]. Eq. (1) has the following special cases:

(i) With $d(t) = 6$, $e(t) = 1$, $l(t) = \epsilon^2$ and $a(t) = b(t) = c(t) = m(t) = n(t) = 0$, Eq. (1) reduces to [36–38]

$$u_t + 6\,u\,u_x + u_{xxx} + \epsilon^2 u_{xxxxx} = 0, \tag{2}$$

which can describe the evolution of solitary waves in fluids with the Bond number less than but close to 1/3 and the Froude number close to 1, and ϵ is a small parameter;

(ii) With $a(t) = 1$, $b(t) = 2$, $d(t) = 3$, $e(t) = -\delta$, $l(t) = \frac{2}{15}$ and $c(t) = m(t) = n(t) = 0$, Eq. (1) takes the form of [39]

$$u_t + uu_{xxx} + 2u_xu_{xx} + 3uu_x - \delta\, u_{xxx} + \frac{2}{15}u_{xxxxx} = 0, \tag{3}$$

which has been derived as a weakly nonlinear long-wave approximation to certain gravity-capillary water-wave problem, and δ is a scaled parameter;

(iii) With $a(t) = b(t) = 15b$, $c(t) = 45b$, $d(t) = 6a$, $e(t) = a$, $l(t) = b$, $m(t) = 0$ and $n(t) = \frac{a^2}{5b}$, Eq. (1) turns out to be the SK equation with a non-vanishing boundary condition [27,40],

$$u_t + a\left(\frac{a}{5b}u + 3u^2 + u_{xx}\right)_x + b(15u^3 + 15uu_{xx} + u_{4x})_x = 0, \tag{4}$$

whose integrability and soliton fusion have been studied [27,40], where a and b are both real constants;

(iv) With the completely-integrable conditions $a(t) = b(t) = \frac{1}{3}c(t)$, $l(t) = \frac{1}{45}c(t)$, $e(t) = \frac{1}{6}\alpha\, c(t)$, $d(t) = \alpha\, c(t)$ and $m(t)$=0 (which will be obtained during the Bell-polynomial process, as seen in Section 3), Eq. (1) becomes

$$u_t + \frac{1}{3}c(t)uu_{xxx} + \frac{1}{3}c(t)u_xu_{xx} + c(t)u^2u_x + \alpha\, c(t)uu_x + \frac{1}{6}\alpha\, c(t)u_{xxx} + \frac{1}{45}c(t)u_{xxxxx} + n(t)u_x = 0, \tag{5}$$

where α is a real constant.

To our knowledge, there has been no work on Eq. (5) as yet. In this paper, we will use the Bell-polynomial approach and Hirota method to investigate Eq. (5). In Section 2, concepts and identities about the Bell polynomials will be briefly presented. In Section 3, we will bilinearize Eq. (5) via the Bell-polynomial approach and derive the N-soliton solutions through the Hirota method. One-, two- and three-solitons will be expressed explicitly. We will analyze the soliton fusion and fission, and influence of the variable coefficients of Eq. (5) in Section 4. Section 5 will give the Bäcklund transformation (BT) and Lax pair through Bell-polynomial mixing variables. Infinitely-many conservation laws will be obtained based on the Bell-polynomial-typed BT in Section 6. Section 7 will be our conclusions.

2. Bell-polynomial preliminary

To start with, we will give basic concepts and identities about the Bell polynomials, which have been shown in Refs. [8–10]. Bell polynomials are defined as

$$Y_{nx}(\omega) \equiv Y_n(\omega_x, \ldots, \omega_{nx}) = e^{-\omega}\,\partial_x^n\, e^{\omega} \qquad (n = 1, 2, \ldots), \tag{6}$$

i.e.,

$$Y_{1x}(\omega) = \omega_{1x}, \quad Y_{2x}(\omega) = \omega_{2x} + \omega_{1x}^2, \quad Y_{3x}(\omega) = \omega_{3x} + 3\,\omega_{1x}\,\omega_{2x} + \omega_{1x}^3, \ldots, \tag{7}$$

where ω is a C^∞ function of the variable x and $\omega_n = \partial_x^n \omega(x)$. Moreover, two-dimensional extensions of the Bell polynomials are introduced as

$$Y_{nx,mt}(\omega) \equiv Y_{n,m}(\omega_{i,j}) = e^{-\omega}\,\partial_x^n\,\partial_t^m\, e^{\omega} \qquad (i = 1, \ldots, n;\ j = 1, \ldots, m), \tag{8}$$

where $\omega_{i,j} = \partial_x^i\,\partial_t^j\,\omega$ with m as another non-negative integer. Two-dimensional binary Bell polynomials are

$$\mathscr{Y}_{nx,mt}(V, U) \equiv Y_{n,m}(\omega_{i,j})\Big|_{\omega_{i,j}=\begin{cases}V_{i,j}, & i+j=\text{odd}\\ U_{i,j}, & i+j=\text{even}\end{cases}}, \tag{9}$$