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A free boundary problem of a predator–prey model with advection in heterogeneous environment

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ABSTRACT

This paper is concerned with a system of reaction–diffusion–advection equations with a free boundary, which arises in a predator–prey ecological model in heterogeneous environment. The evolution of the free boundary problem is discussed. Precisely, we prove a spreading–vanishing dichotomy, namely both prey and predator either survive and establish themselves successfully in the new environment, or they fail to establish and vanishes eventually. Furthermore, when spreading occurs, we obtain an upper bound of the asymptotic spreading speed, which is smaller than the minimal speed of the corresponding traveling wave problem.

1. Introduction

The spatial behavior of populations in homogeneous or heterogeneous environments is a central topic in biology and ecology, and the spreading of the species (subject to laws of diffusion, reaction, advection, and interaction etc.) is a crucial quantity in the study of biological invasions and disease spread. In this paper we consider the following free boundary problem for the Lotka–Volterra type predator–prey modal with an advection term in heterogeneous environments:

$$\begin{cases} u_t - u_{xx} + \beta u_x = u(m(x) - u - av), & t > 0, \quad 0 < x < h(t), \\ v_t - dv_{xx} = v(c - v + bu), & t > 0, \quad 0 < x < h(t), \\ u(t,0) = v(t,0) = 0, & t > 0, \\ u(t,h(t)) = v(t,h(t)) = 0, & t > 0, \\ h'(t) = -\mu[u_x(t,h(t)) + \rho v_x(t,h(t))], & t > 0, \\ u(0,x) = u_0(x), \ \ v(0,x) = v_0(x) & 0 \le x \le h_0 = h(0), \end{cases} \tag{1.1}$$

where β, a, b, c, d, μ and ρ are given positive constants. Biologically, u and v represent, respectively, the spatial densities of prey and predator species that are interacting and migrating in a one dimensional habitat, $m(x)$ accounts for the local growth rate of the prey, the free boundary $x = h(t)$ represents the expanding fronts of the species. Here βu_x is the advection term, which means individuals are confronted with unidirectional drift that drives them out of the system and thus induces decline in population. Such advection term may appear due to some external environmental forces such as water flow

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[22,24], wind [2,3] and gravity [11,23]. The ecological background of free boundary condition in (1.1) can refer to [1,20]. Such kind of free boundary condition has also been used in [12,26,29].

When $\beta = 0$ (i.e., there is no advection in the environment), the analysis of the evolution of the corresponding free boundary problems has been undertaken. Wang [26] studied system (1.1) in homogeneous environment, they proved a spreading–vanishing dichotomy, namely the two species either successfully spread to the entire space as time t goes to infinity and survive in the new environment, or they fail to establish and die out in the long run. Moreover, when spreading successfully, they obtained an estimate to show that the spreading speed (if exists) cannot be faster than the minimal speed of traveling wavefront solutions for the prey–predator model on the whole real line without a free boundary. various two species competition models with free boundaries have been studied in [12,13,29,32,33] and reference therein. Wang and Zhao [30] also studied the similar free boundary problems to (1.1) with double free boundaries in which the prey lives in the whole space but the predator lives in the region enclosed by the free boundary. In particular, in [31], the authors dealt with the higher dimension and heterogeneous environment case.

In the absence of v, problem (1.1) with $\beta = 0$ is reduced to the one phase Stefan problem, which was studied by many authors. Starting from the work of Du and Lin [8], Kaneko and Yamada [17] in homogeneous environment, free boundary problems for the logistic type model, including the higher dimension case, heterogeneous environment case and with seasonal succession, have been extensively studied in [5–7,9,10,17,18,21,25,28,34]. When $\beta > 0$, which means that the species can move up along the gradient of the density (see [14,15] and the reference therein). Due to the appearance of the advection term, new difficulties are introduced mathematically. Currently, there have been few analytical results about single-species free boundary models in the literature. Gu et al. [14–16] studied the corresponding logistic modal with small advection, they discussed how advection term (βu_x) affects the asymptotic spreading speeds when spreading occurs.

Up to our knowledge, the multispecies with advection case in the heterogeneous environment is not treated in the literature. In this paper, we discuss the free boundary problem (1.1) for a reaction–diffusion–advection model in heterogeneous environment. Throughout our paper, the positive function $m(x)$ satisfies $m(x) \in C^1([0,\infty)) \bigcap L^\infty([0,\infty))$ and

$$m_0 := \inf_{x\ge 0} m(x) > 0, \quad m_1 := \sup_{x\ge 0} m(x) \quad \text{and} \quad m_\infty := \lim_{x\to+\infty} m(x). \tag{1.2}$$

The initial functions u_0, v_0 satisfy

$$u_0,\quad v_0 \in C^2([0,h_0]),\quad u_0(0) = v_0(0) = u_0(h_0) = v_0(h_0) = 0,\quad u_0(x),\quad v_0(x) > 0 \quad \text{in} \quad (0,h_0).$$

The main purpose of the present paper is to analyze the effect of the advection term and the heterogeneous environment on the criteria for spreading or vanishing and the asymptotic spreading speeds as spreading occurs. In this sense, the present paper can be regarded as the generalization of [26].

The most challenging issue lies in the discussion of the following corresponding elliptic problem in half line

$$\begin{cases} -du'' + \beta u' = u(f(x) - \lambda u), & 0 < x < \infty, \\ u(0) = 0. \end{cases} \tag{1.3}$$

To overcome the difficulty induced by the advection term, we rewrite (1.3) as

$$\begin{cases} -d\left(e^{-\frac{\beta}{d}x}u'\right)' = e^{-\frac{\beta}{d}x}u(f(x) - \lambda u), & 0 < x < +\infty, \\ u(0) = 0. \end{cases} \tag{1.4}$$

When $\beta = 0$ and $f(x)$ is a monotonous function of x, Wang [26] proved the convergence result

$$\lim_{x\to+\infty} u(x) = \frac{1}{\lambda} \lim_{x\to+\infty} f(x).$$

In contrast to problem (1.3) ($\beta = 0$) discussed in [26], the function $f(x)$ here is influenced by the coefficient $m(x)$ which describes the intrinsic rate of growth and is not monotonous in x. The conclusion in [26] cannot be used directly in this situation. However, by construction monotonous functions and using comparison principle, we also prove the convergence result that $\lim_{x\to+\infty} u(x) = \frac{1}{\lambda}\lim_{x\to+\infty} f(x)$ (see Theorem 2.4). Moreover, the long time behavior of solution and criteria for spreading and vanishing are also obtained. When spreading happens, we also provide the estimate

$$\limsup_{t\to\infty} \frac{h(t)}{t} \le \max\left\{2\sqrt{cd},\ 2\sqrt{m_1} + \beta\right\}$$

to show that the spreading speed (if exists) cannot be faster than the minimal speed of traveling wavefront solutions for the prey–predator model on the whole real line without a free boundary.

By a similar argument as in [8,26], we have the following basic results.

Theorem 1.1. *Problem (1.1) has a unique global solution, and for any $\alpha \in (0, 1)$ and $T > 0$,*

$$(u, v, h) \in [C^{(1+\alpha)/2,1+\alpha}(\overline{D_T})]^2 \times C^{1+\alpha/2}([0,T]),$$

where

$$D_T = \{(t,x) \in \mathbb{R}^2 : t \in (0,T],\ x \in (0,h(t))\}.$$